

CURRICULUM

QA

14

C22

A3

A29

1987

ALTA

510

1987a

gr. 7-9

CURR

CURRGDHT



EX LIBRIS
UNIVERSITATIS
ALBERTÆNSIS

JUNIOR HIGH MATHEMATICS
CURRICULUM GUIDE

1987

DRAFT

Alberta
EDUCATION

UNIVERSITY LIBRARY
UNIVERSITY OF ALBERTA

TABLE OF CONTENTS

ii

GRADE 7 MATHEMATICS

Acknowledgements	ii
Problem Solving	4
Number Systems and Operations	12
Ratio and Proportion	42
Measurement and Geometry	48
Data Management	56
Algebra	62

GRADE 8 MATHEMATICS

Problem Solving	74
Number Systems and Operations	82
Ratio and Proportion	102
Measurement and Geometry	108
Data Management	120
Algebra	126

GRADE 9 MATHEMATICS

Problem Solving	134
Number Systems and Operations	138
Ratio and Proportion	150
Measurement and Geometry	154
Data Management	164
Algebra	170
Algebra—Extended Content	176

ACKNOWLEDGEMENTS

This curriculum guide draft has been prepared with guidance and input provided by a number of individuals. Their suggestions and assistance has proved to be valuable in the development of this document.

ADMINISTRATION AND PRODUCTION

This curriculum document has been prepared within the Junior High Unit, Curriculum Branch, Alberta Education.

JUNIOR HIGH MATHEMATICS ADVISORY COMMITTEE

Mr. Pat McLaughlin (Chairman), Alberta Education
Ms. Linda Cox, Alberta Correspondence School
Mr. Don Darling, Edmonton School District #7
Dr. Tom Kieren, University of Alberta
Mr. Rene Mathieu, Alberta Education
Mr. Garry Canadine, Lethbridge School District #51
Ms. Teresa Scott, Grande Prairie School District #2357
Mr. Bruce Stonell, Stettler School District #1475
Mr. Glen Wilson, County of Minburn #27

JUNIOR HIGH MATHEMATICS DEVELOPMENT COMMITTEE (Ad Hoc)

Mr. John Chapman, High Prairie School Division #48
Ms. Jean Crawford, Calgary Roman Catholic Separate School
District #1
Ms. Myra Hood, Calgary School District #19
Mr. Ken Hutchings, County of Strathcona #20
Mr. Dick Kopan, Calgary School District #19
Mr. Peter Miller, County of Flagstaff #29
Ms. Katie Pallos-Haden, County of Parkland #31
Mr. Leo Sam, County of Smoky Lake #13
Ms. Jan Sparling, Teacher, St. Albert
Ms. Susan Turner, Grande Prairie School District #2357

Jim Brackenbury, Associate Director

Barbara Esdale, Learning Resources Officer
Alexandra Hildebrandt, Learning Resources Officer
Merv Lastiwka, Program Manager
Valerie Perry, Curricular Assistant

Special recognition must go to the following staff members who helped to produce this document:

Kim Blevins, Copy Editor
Lisa McCardle, Editor

Jan Cruickshank, Word Processing

Lin Gray, Word Processing
Denise Hugman, Word Processing
Cheryl Stoochnoff, Word Processing
Janice Yarmuch, Word Processing
Esther Yong, Word Processing

GRADE 7
MATHEMATICS

PLEASE NOTE

The following resources have received basic status and are available from the Learning Resources Distributing Centre:

Student texts for JOURNEYS IN MATH 7, 8

Student texts for HOLT MATH 7, 8, 9

Any other titles mentioned in this guide have not received formal status approval. They are presently under review and cannot, at this time, be obtained through the Learning Resources Distributing Centre.

CURRICULUM GUIDE (DRAFT)

PROBLEM SOLVING

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Demonstrates an understanding of a problem-solving situation.	<p>Problem solving should not be viewed as an isolated activity but rather as a process that is to be an integral part of the teaching philosophy to be used in the development of the other strands. The framework for problem solving should be introduced at the beginning of the year (suggested time – 3 to 5 periods).</p> <p>Brainstorm for a definition and examples. The following ideas should evolve about a problem:</p> <ul style="list-style-type: none"> a) it has no readily apparent solution or the means to the solution is not immediately evident b) it can cause a person to be temporarily perplexed c) it may have no answer, one answer, or more than one answer <p>Teachers must recognize that problem-solving skills are essential for all students and that being perplexed when first encountering a problem is normal. Problems presented to students should be challenging yet solutions must be attainable to insure that students experience success.</p> <p>It is very important for teachers to realize individual student differences in learning; therefore the growth expectation should also vary.</p> <p>Individual needs can often be met by changing the conditions of a problem to make it simpler.</p>		<p>a) The use of calculators in problem solving must be encouraged so that time spent on tedious calculations is decreased and feedback on strategies is faster. Numbers from realistic and relevant situations are less imposing if calculators are used.</p> <p>b) Group work should often be used in problem solving. A student in a group deals with ideas and questions from other members of the group, and this may help each student to progress in developing problem-solving strategies.</p>

Note: (E) = Enrichment
(R) = Remediation

PROBLEM SOLVING

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>1. Demonstrates an understanding of a problem-solving situation. (cont'd)</p> <p>d) it can be of a practical, everyday, personal or social nature as well as of a mathematical nature.</p> <p>(See Journeys in Math 7 TRM, p. 30.)</p>	<p>Manipulatives can also be used to meet individual needs.</p> <p>e.g., Students are given a pile of 21 markers. Two players are involved and take turns removing one, two or three markers. The winner is the player who removes the last marker. The purpose of the game is to develop a strategy to always win. As students continue to work on this they should become more interested in finding a strategy rather than winning. For students who have difficulty with this, decrease the number of markers used or only move one or two markers. Demonstrate how the markers can be grouped and ask students critical questions such as the importance of moving first, and other strategic moves. The game can also be made into a more difficult version to challenge higher ability students. Use two piles and change the rules. Students can take one marker from each pile or one marker from only one pile.</p>		

PROBLEM SOLVING

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>2. Demonstrates a willingness to find a solution to a problem.</p> <p>In order to develop the students' willingness to find solutions, the teacher should:</p> <ol style="list-style-type: none"> create a positive classroom atmosphere that allows students to foster their own ideas and approaches in problem solving be supportive and encourage risk taking in finding solutions encourage students to use creative approaches be willing to accept unconventional solutions, more than one solution, or no solution (where appropriate) challenge students to think critically and justify strategies and solutions be enthusiastic and capable of recognizing the students' willingness and perseverance to solve problems provide appropriate questions for students present problem situations that enable students to gain problem-solving experience that is transferable to other subject areas and everyday life. 	<p>Students who experience difficulty with the complex strategies may find it necessary to use a more concrete approach for a longer period of time and may require more teacher guidance.</p> <p>e.g., A store owner buys candies in bulk bags containing 80 candies each. He re-packages the candies for sale in smaller bags of 12. How many candies are left over when one bulk bag of 80 candies is re-packaged?</p>	<p>Computers may be used to assist in teaching problem solving. Various programs and simulations require the use of particular or various strategies (e.g., Houghton Mifflin MAC, MECC and Sunburst Communications Software).</p> <p>The use of relevant and realistic problems (from sources such as newspapers and magazines) is encouraged because this will increase the interest of students. Students may also be able to contribute their own ideas of problems to solve.</p>	

PROBLEM SOLVING

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Demonstrates a willingness to find a solution to a problem. (cont'd)	<p>Use of a concrete example will help students who experience difficulty with the operation of division.</p> <p>Concrete approaches should be encouraged as long as it is necessary for the student.</p> <p>A teacher should challenge the more capable students by having them not only justify their strategies and solutions but also to consider the possibilities such as:</p> <ol style="list-style-type: none"> <li data-bbox="806 615 848 1037">other strategies and solutions <li data-bbox="864 615 927 1037">"what if?" (change an element of the problem) <li data-bbox="942 615 1005 1037">generalization of rules to other situations. <p>e.g., Using the above candy problem, ask: "How many bulk bags of 80 candies each would the store owner need to re-package so that <u>no candies are left over</u>?"</p>		

PROBLEM SOLVING

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>3. Uses a variety of strategies to solve problems.</p> <p>The following strategies should be developed throughout the various strands of the program and within the problem-solving framework:</p> <ol style="list-style-type: none"> Understanding the problem 	<p>To introduce the strategies of problem solving, an approach such as the following may be used:</p> <p>Choose three non-related but similar problems that can be solved focussing on a strategy (consider that any problem usually requires the application of more than one strategy) such as the strategy of acting out or simulating the problem. The first problem can be a teacher demonstration, the second can be a student trial with teacher guidance, and the third can be student practice.</p> <p>The strategy of acting or simulating the problem can be developed within the problem-solving framework [Understanding the problem, Developing a plan, Carrying out the plan, Looking back] as follows:</p> <p>If six people were in a room and each one shook hands with every other person, how many handshakes were there? (Problem-Solving Challenge for Mathematics, Edmonton: Alberta Education, 1985, p. 42, #29.)</p>	<ul style="list-style-type: none"> • knows the meaning of all the words in the problem • identifies key words • draws a diagram • classifies information as insufficient or extraneous • restates the problem in own words 	

PROBLEM SOLVING

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>a) Understanding the problem (cont'd)</p> <ul style="list-style-type: none"> • uses concrete manipulatives • looks for a pattern • considers an alternative interpretation <p>b) Developing a plan (choosing a strategy)</p> <ul style="list-style-type: none"> • guesses and checks - improves the guess • chooses and sequences • mathematical operations • acts out or simulates the problem • applies a pattern • uses a simpler problem <p>Under the guidance of the teacher, the students may now investigate a similar but non-related problem in a group situation by using the strategy of acting out or simulating. One such problem may be: In how many ways can a committee of two be selected from five people?</p> <p>A practice problem involving an acting out (or using manipulatives) strategy may be as follows:</p> <p>Move three coins on the figure on the left to make it look like the figure on the right. (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, p. 19, #7.7.)</p> <p>The actual use of a manipulative could be very effective for lower ability students.</p> <p><u>Evaluation</u></p> <p>The evaluation of problem solving requires more than grading the solutions to mathematical problems. Continual observation and questioning of students while they are solving problems is essential.</p>			

PROBLEM SOLVING

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>c) Carrying out the plan</p> <ul style="list-style-type: none"> ● applies selected strategies ● presents ideas clearly ● documents the process ● works with care ● works in a group situation <p>d) Looking back</p> <ul style="list-style-type: none"> ● determines if the answer is reasonable ● explains the answer in oral and written form ● states the solution to the problem ● restates the problem with the answer ● considers other possible solutions to the problem ● looks for other ways to solve the problem ● discusses solution process with others. 	<p>In assessing a student's problem-solving skills, the teacher should consider:</p> <ol style="list-style-type: none"> a) willingness to attempt problems b) use of a systematic approach c) selection of appropriate strategies d) efficiency in selection of appropriate strategies e) logical justification of strategies and solutions f) perseverance g) growth in confidence in problem-solving ability h) transfer of problem-solving skills to situations other than mathematics. <p>Evaluation techniques and instruments for problem solving are found in the Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, pp. 7, 8, 52-56.</p>	<p>Problem solving is integrated as an essential part of each strand. Enrichment/Remediation and use of technological devices is therefore outlined in each strand.</p>	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>1. Applies and practises problem-solving skills in new situations.</p> <p>2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations.</p>	<p>The intent is to develop non-routine problem-solving skills in every student. Each student should understand the concepts of the problem-solving framework and develop multiple strategies.</p> <p>Specific problem-solving techniques and skills will be developed with each objective.</p> <p>An equal emphasis should be placed on the various strategies for computing. Single-digit basic facts should be drilled on a regular basis through activities such as timed challenges or games. Paper-and-pencil strategies should be used to develop an understanding of sub-concepts such as re-grouping, borrowing or place value. Long and tiresome paper-and-pencil drill is discouraged.</p> <p>Estimation should be done on a daily basis. Recognition of appropriate situations for estimates, determining how precise an estimate should be for a given situation, and knowing when a computed answer is possible, are among skills to be emphasized.</p>		<p>Students must be taught how to use calculator features after understanding of a concept is developed. Memory keys (order of operations) and percent keys operate on fundamental mathematics concepts. Students should be encouraged to discover these concepts. In addition, students must be taught how to interpret results. Knowing how to find a remainder or the repeating period of a rational number are implicit learning outcomes.</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations. (cont'd)	<p>Mental computation involves using natural and easy strategies to compute exact answers. Strategies should be identified and shared as they evolve.</p> <p>Calculators should be used to develop understanding to investigate patterns, and to perform tedious computations that do not enhance understanding.</p>		

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions											
<p>A. Whole Numbers</p> <p>1. Maintains previously developed skills with whole numbers (place value, standard and expanded forms, adding, subtracting, multiplying and dividing whole numbers).</p>	<p>Use a diagnostic tool to determine student proficiency with each of the operations.</p> <p>Facility with basic facts through activities such as:</p> <p>(i) "Mad Minute" This activity is designed to maintain basic skills in a timed situation. Encourage students to complete as many questions as quickly and accurately as possible.</p> <p>(ii) Drill using computer software Holt Disks 3 and 5, MAC Courseware.</p> <p>(iii) Engage students in frequent estimation activities, e.g., place blocks/rods on overhead. Have students estimate the number. Students demonstrate their answers with cardboard tiles (0-9). Narrow the range of possible answers by giving clues (odd #, perfect square, palindrome, etc.).</p> <p>(R) To reinforce place value encourage the use of manipulatives. Each student should have his/her own material to handle. Make sure that proportional aids are used (these are aids that have a size difference). e.g., These aids work effectively on the proportional level:</p> <p><u>Exchange this</u> <u>for this</u></p> <table> <tbody> <tr> <td>10 beans</td> <td>1 bean stick</td> </tr> <tr> <td>10 white</td> <td>1 orange</td> </tr> <tr> <td>Cuisenaire rods</td> <td>Cuisenaire rod</td> </tr> <tr> <td>10 single beads</td> <td>1 string of 10 beads</td> </tr> <tr> <td>10 squares cut</td> <td>1 10-square strip from graph paper</td> </tr> <tr> <td>from graph paper</td> <td>of graph paper</td> </tr> </tbody> </table> <p>Work slowly with the aid. Once students understand the concept you can try using changeable aids. At this point you can move on to the concept at a more abstract level (adapted from <i>Arithmetic Teacher</i>, Vol. 32, #1, September 1984, p. 21).</p> <p>e.g., Using blocks/rods on the overhead to show the number 625. Have students estimate.</p> <p>Students demonstrate the number with cardboard tiles. Continue giving clues until students display the correct answer.</p>	10 beans	1 bean stick	10 white	1 orange	Cuisenaire rods	Cuisenaire rod	10 single beads	1 string of 10 beads	10 squares cut	1 10-square strip from graph paper	from graph paper	of graph paper
10 beans	1 bean stick												
10 white	1 orange												
Cuisenaire rods	Cuisenaire rod												
10 single beads	1 string of 10 beads												
10 squares cut	1 10-square strip from graph paper												
from graph paper	of graph paper												

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology																																																		
A. Whole Numbers	<p>1. Maintains previously developed skills with whole numbers (place value, standard and expanded forms, adding, subtracting, multiplying and dividing whole numbers). (cont'd)</p>	<p>(E) Patterns in the Multiplication Tables.</p> <p>Display a 10×10 hundred chart with stencils cut from tagboard which allow only one set of multiples to be displayed. Ask students what they see (Fig. 1). Continue doing the same activity with 3s, 4s, 5s, 6s, 7s, 8s, 9s. To allow all students to participate, have them begin quietly and independently. Have them write notes to themselves and then share ideas. Have students complete a 3s table. Discuss the pattern that emerges. Ask students what happens when 2s and 3s are combined? (6s)</p>	<p>Guess and check strategy can be useful when reinforcing skills/operations with whole numbers.</p> <p>e.g., Karen sold \$50 worth of tickets. Student tickets were \$2 and adult tickets were \$3. If she sold 10 student tickets, how many adult tickets did she sell?</p> <ol style="list-style-type: none"> 1) Student guess. 2) Ask questions for understanding of the problem: What are you finding? Price of tickets? Number of tickets sold? etc. 3) Ask questions for solving (hints). 4) Follow up – discuss/focus on the strategy students used – the solution and alternative solutions. <p>Fig. 1. Multiples</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>2</td><td>4</td><td>6</td><td>8</td><td>10</td></tr> <tr> <td>12</td><td>14</td><td>16</td><td>18</td><td>20</td></tr> <tr> <td>22</td><td>24</td><td>26</td><td>28</td><td>30</td></tr> <tr> <td>32</td><td>34</td><td>36</td><td>38</td><td>40</td></tr> <tr> <td>42</td><td>44</td><td>46</td><td>48</td><td>50</td></tr> <tr> <td>52</td><td>54</td><td>56</td><td>58</td><td>60</td></tr> <tr> <td>62</td><td>64</td><td>66</td><td>68</td><td>70</td></tr> <tr> <td>72</td><td>74</td><td>76</td><td>78</td><td>80</td></tr> <tr> <td>82</td><td>84</td><td>86</td><td>88</td><td>90</td></tr> <tr> <td>92</td><td>94</td><td>96</td><td>98</td><td>100</td></tr> </table>	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100
2	4	6	8	10																																																	
12	14	16	18	20																																																	
22	24	26	28	30																																																	
32	34	36	38	40																																																	
42	44	46	48	50																																																	
52	54	56	58	60																																																	
62	64	66	68	70																																																	
72	74	76	78	80																																																	
82	84	86	88	90																																																	
92	94	96	98	100																																																	

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology																																																																																																			
A. Whole Numbers <ul style="list-style-type: none"> 1. Maintains previously developed skills with whole numbers (place value, standard and expanded forms, adding, subtracting, multiplying and dividing whole numbers). (cont'd) 	3s	<tr><td>3</td><td>6</td><td>9</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>12</td><td>15</td><td>18</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>21</td><td>24</td><td>27</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td>33</td><td>36</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td>42</td><td>45</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>51</td><td>54</td><td>57</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td>63</td><td>66</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td>72</td><td>75</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>81</td><td>84</td><td>87</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td>93</td><td>96</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>	3	6	9								12	15	18								21	24	27									33	36									42	45								51	54	57									63	66									72	75								81	84	87									93	96							
3	6	9																																																																																																				
12	15	18																																																																																																				
21	24	27																																																																																																				
	33	36																																																																																																				
	42	45																																																																																																				
51	54	57																																																																																																				
	63	66																																																																																																				
	72	75																																																																																																				
81	84	87																																																																																																				
	93	96																																																																																																				

 (Arithmetic Teacher, Vol. 32, #7. March 1985, pp. 36-37.) |

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>A. Whole Numbers</p> <p>2. Understands properties of number operations and uses properties and relationships to perform mental computations (e.g., associative, commutative, distributive).</p>	<p>The intention here is to develop student understanding of the properties through their indirect use.</p> <p>Encourage students to do many "mental exact" computations and then explain the properties.</p> <p>e.g.,</p> <ol style="list-style-type: none"> 1. $23 + 28 + 7 + 2 =$ 2. $16 + 9 + 4 =$ 3. $2 \times 18 \times 10 =$ 4. $16 \times 12 = 10(12) + 6(12) =$ 5. add mentally from left to right <p style="text-align: right;">236 <u>587</u></p>	<p>(R) Timed drill activities on mental computations can be used to increase student skills.</p> <p>It is important that students understand the properties and relationships rather than define them. Students can demonstrate their understanding by their ability to verbalize.</p>	

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers	<p><u>Pattern</u></p> <p>Using a calculator or computer direct students to divide a given number by a set of numbers that become smaller (approach 0).</p> <p>e.g., $10, 5, 2, 1, \frac{1}{2}, \frac{1}{10}, \frac{1}{100}$</p> <p>Ask students to predict what would happen if they divided by 0. Check on a calculator. Does an "Error" message appear? Why?</p>	<p>(R) What does division mean? (Repeated subtraction.)</p> <p>Use concrete manipulatives to count how many times, for example, a group of 2 can be subtracted from 6. Then ask "how many times can 0 blocks be removed from 6?" Discussion should lead students to the conclusion that the question itself is meaningless.</p>	

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>A. Whole Numbers</p> <p>4. Writes the value of a power (whole number base and exponent).</p>	<p>Use blocks to demonstrate/investigate patterns.</p> <p>Teachers may wish to use alternative concrete manipulatives such as sticks, beans, coloured squares or grid paper.</p> <p>$2^1 = 2$ (blocks) $2^2 = 4$ (blocks) $2^3 = 8$ (blocks) $2^4 = 16$ (blocks) etc.</p> <p>$\square \quad \square \quad \square \quad \square \quad ?$ $\square \quad \square \quad \square \quad \square \quad \square \quad ?$ $2^1 \quad 2^2 \quad 2^3 \quad 2^4$</p>	<p>(R) Extended use of blocks.</p> <p>(E) Reverse the procedure. Look at a variety of patterns leading to an understanding of $x^0 = 1$ at a concrete level.</p> <p>e.g., $2^4, 2^3, 2^2, 2^1, 2^0$ $16 \quad 8 \quad 4 \quad 2 \quad ?$</p> <p>e.g., $7^4, 7^3, 7^2, 7^1, 7^0$ $2401 \quad 343 \quad 49 \quad 7 \quad ?$</p>	<p>Using calculators and computers develop the investigation further. (Arithmetic Teacher, Vol. 34, #6, February 1987, p. 36.)</p> <p>Write the values for the powers.</p> <p>The following activity encourages the development of guess and check.</p> <p>Guess the number that goes in the box \square.</p> <p>Check with calculator; continue until the correct number is found</p> <p>$\square^3 = 12167$ $\square^3 = 704969$ $\square^4 = 1419857$ $\square^4 = 16807$</p> <p>Challenge students to find the number in 3 tries? 4? 5?</p>

NUMBER SYSTEMS AND OPERATIONS

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>A. Whole Numbers</p> <p>4. Writes the value of a power (whole number base and exponent). (cont'd)</p>	<p>e.g.,</p> <p><input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> 27 <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> ?</p> <p><input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> 9 <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> triple <input type="checkbox"/> <input type="checkbox"/> it 32 33 34</p> <p><input type="checkbox"/> <input type="checkbox"/> it</p> <p>31</p>		<p>Students can continue developing the concept to a factor of 10. They should be able to extend their understanding to include larger numerical factors.</p>

NUMBER SYSTEMS AND OPERATIONS

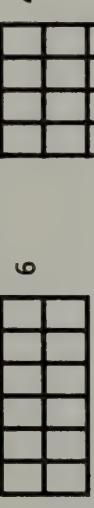
Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>A. Whole Numbers</p> <p>5. Applies the rules for the order of operations to evaluate expressions.</p>	<p>The intent is to pose a problem involving the order of operations. Students should use calculators and computers to find a solution for $67 - 8 \times 3$.</p> <p>As two solutions emerge, focus discussion on order of operations.</p> <p>The following activity will reinforce this skill:</p>	<p>(E) Calculator.</p> <p>Challenge students to develop a method which uses the memory function to solve problems using the order of operations.</p> <p>e.g., $67 - 8 \times 3 \leftrightarrow MC$ $8 \times 3 = M + 67 - MR = 43$</p>	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 6. Recognizes prime numbers (limit: primes to 50).	<p>Students should make all possible rectangles using a given prime number.</p> <p>This concept is fully developed for factors and can be extended to prime numbers.</p> <p>e.g.,</p>	<p>17</p> 	<p>(R) Reinforcement.</p> <p>Continue use of manipulatives: e.g., Sieve of Eratosthenes.</p> <p>A method of making a list of prime numbers was devised by an ancient Greek scholar, Eratosthenes, in the third century B.C. The method consists of taking a 100 chart and circling the first five prime numbers, 2, 3, 5, 7, and 11. Then all numbers divisible by 2, 3, 5, 7, and 11 are crossed out.</p> <p>The prime numbers will not be crossed out.</p> <p>Students will discover that only one possible example exists.</p> <p>(E) This exercise can be completed using a utility program or with pencil computations.</p> <p>Computer Program:</p> <pre> 2 PRINT "PRIME NUMBERS < 50" 5 PRINT "2" 10 FOR X = 2 TO 50 20 FOR Y = 2 TO SQR(X) 40 IF X/Y = INT(X/Y) THEN 50 NEXT Y 60 PRINT X " "; 70 NEXT X </pre> <p>(1) 10 consecutive numbers with two or more digits – what is the maximum number of primes?</p> <p>(2) Set of 10 consecutive three-digit numbers with four? With only three? With only two? None? (Answer: 100-109; 130-139; 160-169; 110-119; 120-120)</p> <p>(3) 11 and 101 are prime. Is 1001? Check out 10 001, 100 001 and 1 000 001 on the computer.</p> <p>(Adapted from Arithmetic Teacher, Vol. 34, #5, January 1987.)</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>A. Whole Numbers</p> <p>7. Lists the set of factors for whole numbers up to 200.</p>	<p>Develop a method for finding factors at a <u>concrete</u> and <u>semi-concrete</u> level <u>before</u> <u>formal</u> presentation.</p> <p>e.g., Use tiles to develop factors of a given number. Using blank cardboard tiles, students should make all possible rectangles with an area of 12 square units. Draw and label each rectangle as it is formed.</p>  	<p>(E) Repeated division can be used to find factors of a number, as well as reducing a fraction to lowest terms and giving the L.C.M.</p> <p>e.g., $\begin{array}{r} 2(24 \quad 36) \\ \quad 2(12 \quad 18) \\ \quad \quad 3(6 \quad 9) \\ \quad \quad \quad 2 \quad 3 \end{array}$</p> <p>$\text{L.C.M.} = 2 \times 2 \times 3 \times 2 \times 3 = 72$</p> <p>Reduce 24/36</p> $\begin{array}{r} 2(24 \quad 36) \\ \quad 2(12 \quad 18) \\ \quad \quad 3(6 \quad 9) \\ \quad \quad \quad 2 \quad 3 \end{array}$ <p>$\frac{24}{36} = \frac{2}{3}$</p>	<p>The dimensions of the various rectangles reveal the factors of the given number (12).</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>A. Whole Numbers</p> <p>7. Lists the set of factors for whole numbers up to 200. (cont'd)</p>	<p>Students may develop or use a simple program which generates a set of factors.</p> <p>(Arithmetic Teacher, Vol. 34, #5, January 1987, p. 36.)</p>	<p><u>Basic Program</u></p> <pre> 100 PRINT "WHAT IS THE NUMBER"; 110 INPUT N 120 PRINT "THE FACTORS ARE" 130 FOR K = 1 TO N 140 LET Q1 = N/K 150 LET Q2 = INT(N/K) 160 IF Q1 = Q2 THEN PRINT K 170 NEXT K 180 END </pre> <p><u>LOGO Program</u></p> <pre> TO FACTOR :N MAKE "K 0 PRINT (THE FACTORS ARE) CHECK :N :K END TO CHECK :N :K REPEAT :N[MAKE "K :K + 1 TEST REMAINDER :N :K = 0 IF TRUE THEN PRINT :K END </pre>	<p>See elective suggestions.</p>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology																									
<p>A. Whole Numbers</p> <p>8. Expresses a number as a product of its prime factors.</p>	<p>Encourage the use of calculators and/or factor trees.</p> <p>Repeated division on a calculator or using the repeated division method can be helpful. Students must remember that only primes can be used.</p> <p>With a calculator, guess and check can be used to see what primes go into a given number.</p>	<p>(E) Have students look at numbers whose prime factorization consists of one prime factor; e.g., $16 = 2 \times 2 \times 2 \times 2 = 2^4$. All of the factors of 16 can be expressed as $1, 2^1, 2^2, 2^3$ and 2^4. What is the relationship? (Five factors \rightarrow one more than the exponent.) Have students experiment with other numbers.</p>	<p>Problem Solving: At a junior high school there are 1000 students and 1000 lockers. The lockers are numbered in order from 1 to 1000. A student entered the building and opened every locker. A second student closed every locker having an even number. A third student changed every third locker, closing those that were open and opening those that were closed. A fourth student changed the fourth locker, and so on. This continued until all 1000 students passed through the locker room. What was the position of locker #1000?</p> <p>Strategies: Reduce to a simpler problem; e.g., What if there were only 20 students and 20 lockers?</p> <p>Make a chart to organize the data.</p> <table border="1" data-bbox="994 285 1226 429"> <thead> <tr> <th data-bbox="994 285 1041 429">Students</th> <th data-bbox="1041 285 1088 429">1</th> <th data-bbox="1088 285 1135 429">2</th> <th data-bbox="1135 285 1183 429">3</th> <th data-bbox="1183 285 1226 429">4</th> </tr> </thead> <tbody> <tr> <td data-bbox="994 429 1041 486">Locker #</td> <td data-bbox="1041 429 1088 486">1</td> <td data-bbox="1088 429 1135 486">0</td> <td data-bbox="1135 429 1183 486"></td> <td data-bbox="1183 429 1226 486"></td> </tr> <tr> <td data-bbox="994 486 1041 543">1</td> <td data-bbox="1041 486 1088 543">2</td> <td data-bbox="1088 486 1135 543">0</td> <td data-bbox="1135 486 1183 543"></td> <td data-bbox="1183 486 1226 543"></td> </tr> <tr> <td data-bbox="994 543 1041 598">2</td> <td data-bbox="1041 543 1088 598">3</td> <td data-bbox="1088 543 1135 598">0</td> <td data-bbox="1135 543 1183 598">C</td> <td data-bbox="1183 543 1226 598"></td> </tr> <tr> <td data-bbox="994 598 1041 655">3</td> <td data-bbox="1041 598 1088 655">4</td> <td data-bbox="1088 598 1135 655">0</td> <td data-bbox="1135 598 1183 655">C</td> <td data-bbox="1183 598 1226 655">0</td> </tr> </tbody> </table> <p>Look for a pattern.</p> <p>Extension: How many students changed the position of locker #600?</p>	Students	1	2	3	4	Locker #	1	0			1	2	0			2	3	0	C		3	4	0	C	0
Students	1	2	3	4																								
Locker #	1	0																										
1	2	0																										
2	3	0	C																									
3	4	0	C	0																								

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>A. Whole Numbers</p> <p>8. Expresses a number as a product of its prime factors. (cont'd)</p>	<p>In each chart, what are the next five numbers after 99?</p> <p>In our number system, base ten, 9 is one less than 10. How does this fact make the pattern diagonal on the chart?</p>	<p>(Arithmetic Teacher, Vol. 32, #7, March 1985, p. 39.)</p>	

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 9. Uses a calculator or microcomputer to generate a set of multiples of a given number.	<p>Reinforce concept of multiples with a simple verbal activity.</p> <p>e.g., count by 2's count by 3's count by 5's</p> <p>Have them investigate ways they can generate sets of multiples on a calculator.</p>	<p>(E) Use the computer program to generate <u>common</u> multiples for two or more numbers.</p> <p>(R) Have students <u>use</u> utility programs to generate multiples.</p> <p>This is a simple BASIC program that will do this:</p> <pre> 100 REM FIND THE LEAST COMMON MULTIPLE 110 READ A, B, C 120 LET X = A 130 IF INT (X/A) = X/A THEN 160 140 LET X = X + 1 150 GO TO 130 160 IF INT (X/B) = X/B THEN 190 170 LET X = X + 1 180 GO TO 130 190 IF INT (X/C) = X/C THEN 220 200 LET X = X + 1 210 GO TO 130 220 PRINT "THELCM OF"; A; B; C; "IS"; X 230 GO TO 110 240 DATA 250 DATA 260 END RUN </pre>	<p>Include in development of the lesson.</p> <p>Problem</p> <ol style="list-style-type: none"> 1) There are two sizes of tables in a banquet hall. One size seats exactly 5 people and the other size seats exactly 8 people. At tonight's banquet, exactly 79 people will be seated at less than one dozen tables, and there will be no empty places. How many tables of each size will there be? 2) The members of a flag squad wanted to arrange themselves into rows with exactly the same number of squad members in each row. They tried rows of 2, 3, and 4, but there was always one squad member missing. Finally they were able to arrange themselves into rows with exactly 5 in each row. What is the least number of members in the flag squad? <p>(pp. 90-91, <u>Creative Problem Solving</u>, G. Lenchner, Houghton Mifflin Company)</p>

NUMBER SYSTEMS AND OPERATIONS

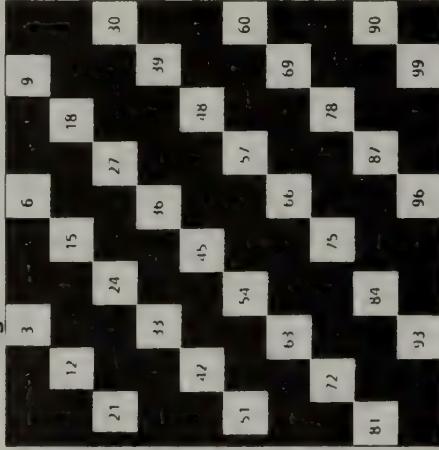
Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology																																																		
<p>A. Whole Numbers</p> <p>10. Determines whether a number is divisible by 2, 3, 5, 6, 9 or 10.</p>	<p>Use the calculator to reinforce divisibility rules.</p> <p>Teachers may also wish to review divisibility rules with the use of charts.</p>	<p>(R) Form divisibility charts. Students can make posters to show which tables have ending rules or sum of digit rules or one that displays 2s, 5s, and 10s tables and another that displays 3s and 9s tables. Display the posters with questions underneath.</p> <p>Fig. 4. Divisibility rules: Ending rules</p> <table border="1" data-bbox="724 629 1048 1010"> <tbody> <tr> <td>2</td> <td>4</td> <td>6</td> <td>8</td> <td>10</td> </tr> <tr> <td>12</td> <td>14</td> <td>16</td> <td>18</td> <td>20</td> </tr> <tr> <td>22</td> <td>24</td> <td>26</td> <td>28</td> <td>30</td> </tr> <tr> <td>32</td> <td>34</td> <td>36</td> <td>38</td> <td>40</td> </tr> <tr> <td>42</td> <td>44</td> <td>46</td> <td>48</td> <td>50</td> </tr> <tr> <td>52</td> <td>54</td> <td>56</td> <td>58</td> <td>60</td> </tr> <tr> <td>62</td> <td>64</td> <td>66</td> <td>68</td> <td>70</td> </tr> <tr> <td>72</td> <td>74</td> <td>76</td> <td>78</td> <td>80</td> </tr> <tr> <td>82</td> <td>84</td> <td>86</td> <td>88</td> <td>90</td> </tr> <tr> <td>92</td> <td>94</td> <td>96</td> <td>98</td> <td>100</td> </tr> </tbody> </table>	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100	<p>Develop a program which tests for divisibility.</p> <p><u>Problem</u></p> <ol style="list-style-type: none"> The pages of a certain book are numbered consecutively from 1 to 500. How many page numbers meet each of the following conditions? <ol style="list-style-type: none"> The page numbers contain the digit 5 and are divisible by 5. The page numbers contain the digit 5 but are not divisible by 5. The page numbers do not contain the digit 5 but are divisible by 5. Replace the missing digit so that the resulting number is divisible by 9. <ol style="list-style-type: none"> 1 4 5 6 □ 28 □ 6 4 9 □ 5 4 3 2 1 <p>Replace the missing digit so that the resulting number is divisible by 3.</p>
2	4	6	8	10																																																	
12	14	16	18	20																																																	
22	24	26	28	30																																																	
32	34	36	38	40																																																	
42	44	46	48	50																																																	
52	54	56	58	60																																																	
62	64	66	68	70																																																	
72	74	76	78	80																																																	
82	84	86	88	90																																																	
92	94	96	98	100																																																	

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>A. Whole Numbers</p> <p>10. Determines whether a number is divisible by 2, 3, 5, 6, 9 or 10. (cont'd)</p>		<p>10 20 30 40 50 60 70 80 90 100</p>	<p>What tables are these? Look at the digits in the units place. Do you see a pattern? How can you recognize the numbers that belong in these tables? In each of these tables, what are the next five numbers after 100? Our number system is base ten; in base ten the 2s, 5s, and 10s tables have ending rules. How are 2, 5, and 10 related to the number 10?</p>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>A. Whole Numbers</p> <p>10. Determines whether a number is divisible by 2, 3, 5, 6, 9 or 10. (cont'd)</p>	<p>Fig. 5. Divisibility rules: Sum of digits</p> 	<p>Fig. 5. Divisibility rules: Sum of digits</p>	<p>What tables are these? Add the digits in each number. What pattern do you see in the answers?</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>B. Decimals</p> <p>1. Maintains previously developed skills with decimal numbers (place value, expanded and standard forms, adding, subtracting, multiplying and dividing decimal numbers).</p>	<p>Assess student proficiency in basic operations (see Objective # 1, whole numbers for drill and practice activities).</p> <p>Encourage the use of varying computational strategies when reviewing basic operations.</p> <p>(Game: Adapted from Arithmetic Teacher, Vol. 32, #6, February 1985, p.56.)</p> <p><u>Calculating by Teams</u></p>	<p>(R) Use base 10 blocks to reinforce place value basic concepts.</p> <p>(R) "Mad Minute" or "Math Warm-ups" can be used to increase student proficiency.</p> <p>(E) Discuss division of decimal numerals using a calculator. Ask students how they express the remainder when it is given in decimal form. How many are left over? Students could explore repeating remainders.</p> <p>(R) Fig. 4 $3\overline{)6.93}$</p>	<p>Houghton Mifflin MAC 7, Disk A.</p> <p>Objective: To practise computation with fractions, decimals, and percentages emphasizing speed and accuracy.</p> <p>Directions:</p> <ol style="list-style-type: none"> 1. On an $8\frac{1}{2}'' \times 11''$ sheet of paper, create six questions in large writing like these (for decimals): <ol style="list-style-type: none"> a. $2.45 + 6.8 = \square$ b. $\square - 0.027 = \bigcirc$ c. $5.1 + 7 + \bigcirc = \Delta$ d. $\Delta - 10.023 = \square$ e. $\square + 6.01 = \triangle$ f. $2.69 + \triangle = \underline{\hspace{2cm}}$ <p>Reproduce the sheets so that you have as many as the number of rows of six students.</p> <ol style="list-style-type: none"> 2. Cut the activity sheets into strips containing one problem.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>B. Decimals</p> <p>1. Maintains previously developed skills with decimal numbers (place value, expanded and standard forms, adding, subtracting, multiplying and dividing decimal numbers). (cont'd)</p>	<p>3. Distribute the strips in order (a-f) down the row. When each student has a problem strip, give the signal to begin. The student with problem strip b cannot do the problem until the number that goes into the \square is passed on from the student with problem strip a.</p> <p>4. The student with problem strip f brings the "final answer" to the teacher for checking. Students are encouraged to double-check their work and can signal at any time that a different answer is being passed back because of an error. The first row to submit the correct final answer is awarded one point for each player.</p>	<p><i>Going further</i></p> <ol style="list-style-type: none"> 1. Have students make up problem strips for future relays. The teacher will eventually have a nice supply for classes in future years. 2. Make up strips with only decimals, fractions, and so on, or combinations of operations. This activity can be adapted for younger students by creating strips with easier problems. 	

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>B. Decimals</p> <p>1. Maintains previously developed skills with decimal numbers (place value, expanded and standard forms, adding, subtracting, multiplying and dividing decimal numbers). (cont'd)</p>	<p>3. Alternate methods for passing out strips to keep the competition fair (e.g., vertical or horizontal rows, front to back or back to front). Post a cumulative record of points earned.</p> <p>(From the file of Leona Burke Worth, Township of Ocean School District, Oakhurst, NJ 07755.)</p> <p>Write the digits in the boxes to make the largest and the smallest answers. Use each digit only once.</p>	<p>Largest Answer Smallest Answer</p> <p>2. 9, 3, 1, 6 $\square \square . 7 + \square . \square$ $\square \square . 7 + \square . \square$ 3. 3, 4, 7, 3 $6 \square . \square - \square . \square$ $6 \square \square - \square . \square$ 4. 8, 4, 7, 0 $3.4 \square \square - 2. \square \square$ $3.4 \square \square - 2. \square \square$ 5. 2, 8, 6, 1 $4. \square \square \times \square \square$ $4. \square \square \times \square \square$</p> <p>(Arithmetic Teacher, Vol. 34, #7, March 1987, p. 31.)</p>	

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology										
<p>B. Decimals</p> <p>2. Compares and orders decimal numbers.</p>	<p>Fold a strip of paper repeatedly to form a decimal number line, giving students a tool to order and compare simple decimals.</p> <p>e.g.,</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">0.25</td> <td style="text-align: center;">0.5</td> <td style="text-align: center;">0.75</td> </tr> <tr> <td style="text-align: center;">0.125</td> <td style="text-align: center;">0.250</td> <td style="text-align: center;">0.375</td> <td style="text-align: center;">0.500</td> <td style="text-align: center;">0.625</td> <td style="text-align: center;">0.750</td> <td style="text-align: center;">0.875</td> </tr> </table> <p>Game</p> <p>Materials: Decimals written on $8\frac{1}{2}'' \times 11''$ sheets of paper (one number on each sheet).</p>	0.25	0.5	0.75	0.125	0.250	0.375	0.500	0.625	0.750	0.875	<p>(E) Extend the folding experiment and have students address the concept of infinity between two points.</p> <p>Teachers may wish to develop dot to dot games which order decimals from least to greatest.</p>	<p>Develop or use a program to order/compare decimals.</p> <p>(Arithmetic Teacher, Vol. 34, #7, March 1987.)</p>
0.25	0.5	0.75											
0.125	0.250	0.375	0.500	0.625	0.750	0.875							

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions Integration of Problem Solving and Technology
<p>B. Decimals</p> <p>2. Compares and orders decimal numbers. (cont'd)</p>	<p>5. The team size can be increased to make the game more difficult.</p> <p>This ability can be modified for lower grades to teach several concepts involving ordering.</p>	<p>(R) Give students a problem in which they would have to round the answer to increase their understanding.</p> <p>(E) Plane flying from Gander to London. Halfway across the ocean the plane develops problems. Should the plane go back or continue the journey?</p> <p>(See Journeys in Math 7 TRM, p. 38.)</p> <p>(E) See Journeys in Math 7 TRM, p. 39.</p> <p>(See Journeys in Math 7 TRM, p. 38.)</p> <p>Give real life examples to demonstrate the need for rounding decimals.</p> <p>e.g., 1 litre of gas @ 33.7¢ 25 litres of gas ? Could you pay for this and get exact change back ?</p> <p>Encourage estimation and the use of calculator to check answers.</p> <p>Have students think of other examples in everyday situations where decimals are rounded.</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>C. Fractions</p> <p>1. Maintains previously developed skills with fractions (concept of a fraction, need for fractional numbers, equivalent fractions, basic fractions) at a concrete level.</p>	<p>Teachers may have students use strips of adding machine tape as a measuring unit and have them measure various objects and find the need to subdivide their unit, and discuss the concept of a fraction.</p> <p>Students could fold their measuring unit into 2, 4, 8, 3, 6, etc., and label as directed.</p>	<p>(E) Develop fraction dominos to reinforce equivalent fractions.</p> <p>(R) Continue to allow students to use fraction tapes/multiple boards until the concept is fully understood.</p>	<p>MAC 7, Disk B.</p>

Fraction Tape

0	1
0	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{2}{2}$
0	$\frac{3}{4}$
$\frac{1}{4}$	$\frac{4}{4}$

Using this unit they should identify equivalent fractions.

Students may also use multiple boards to identify equivalent fractions.

e.g., $1/3 = ?$

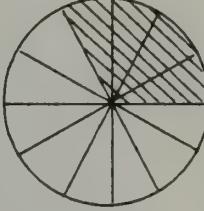
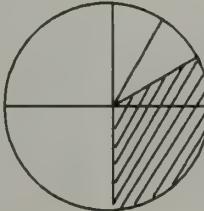
1 2 3 4 5 6 7 8 9 10

3 6 9 12 15 18 21 24 27 30

e.g., $1/3 = 3/9$, etc.

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>C. Fractions</p> <p>2. Identifies mixed numbers and improper fractions and converts from one to the other.</p>	<p>Students should develop this concept in pictorial form.</p> <p>Pizza</p>   <p>Divide amongst three people equally. Each person receives $1\frac{1}{3}$ pieces.</p> <p>Divide amongst three people equally. Each person receives $\frac{4}{3}$ pieces.</p> <p>After some examples, students should develop the rules for conversion.</p>	<p>(R) Encourage the use of manipulatives. Paper circles, pizzas, chocolate bars can be quite effective to illustrate this concept.</p>	<p>If students experience difficulty with this at the pictorial level it can be explained to them using concrete manipulatives; e.g., fraction tape. (See Journeys in Math 7 TRM, p. 172.) This concept should be developed until students can carry out the operation at an abstract level.</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology																																								
<p>C. Fractions</p> <p>3. Orders fractional numbers.</p>	<p>Use the measuring unit from Objective #1 to order fractional numbers (limit to those fractions shown). Use multiple boards to compare fractions.</p> <p>e.g., compare $1/2$ and $4/6$</p> <p>Form $1/2$</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td><td>14</td><td>16</td><td>18</td><td>20</td></tr> </table> <p>Form $4/6$</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>4</td><td>8</td><td>12</td><td>16</td><td>20</td><td>24</td><td>28</td><td>32</td><td>36</td><td>40</td></tr> <tr><td>6</td><td>12</td><td>18</td><td>24</td><td>30</td><td>36</td><td>42</td><td>48</td><td>54</td><td>60</td></tr> </table> <p>Since $3/6 < 4/6$ then $1/2 < 4/6$</p>	1	2	3	4	5	6	7	8	9	10	2	4	6	8	10	12	14	16	18	20	4	8	12	16	20	24	28	32	36	40	6	12	18	24	30	36	42	48	54	60	<p>(E) Teachers may wish to use fractions with unusual denominators wherein the order is not as obvious.</p>	<p>Some students may write a utility program and others may use one to order fractions. This will also generate a group discussion on conversion from fractions to decimals.</p> <p>MAC Courseware 7, Disk B.</p>
1	2	3	4	5	6	7	8	9	10																																		
2	4	6	8	10	12	14	16	18	20																																		
4	8	12	16	20	24	28	32	36	40																																		
6	12	18	24	30	36	42	48	54	60																																		
<p>4. Uses concrete manipulatives to demonstrate the addition and subtraction of fractions with and without common denominators.</p>	<p>While many students are able to use a "rules" approach to operating with fractions, there is much evidence to indicate that fewer students understand the operation.</p> <p>A number of manipulatives may be used to demonstrate addition and subtraction. Cuisenaire rods - Teachers and students should be familiar with the relationships among the various lengths and colours of the rods. (Instructions and activities are usually included when Cuisenaire rods are purchased.)</p>	<p>(R) A multiplication table produces a multiple board for fractions, from which equivalent fractions can be determined.</p> <p>e.g.,</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>...</td></tr> <tr><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td><td>...</td></tr> <tr><td>3</td><td>6</td><td>9</td><td>12</td><td>15</td><td>18</td><td>...</td></tr> <tr><td>4</td><td>8</td><td>12</td><td>16</td><td>...</td><td></td><td></td></tr> </table>	1	2	3	4	5	6	...	2	4	6	8	10	12	...	3	6	9	12	15	18	...	4	8	12	16	...			<p>"Fraction Challenge" MAC Courseware 7, Disk B.</p> <p>$\frac{1}{2} + \frac{2}{3} = \frac{7}{6}$</p>												
1	2	3	4	5	6	...																																					
2	4	6	8	10	12	...																																					
3	6	9	12	15	18	...																																					
4	8	12	16	...																																							

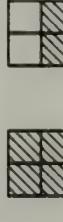
NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>C. Fractions</p> <p>4. Uses concrete manipulatives to demonstrate the addition and subtraction of fractions with and without common denominators. (cont'd)</p>	<p>Fraction bars or fraction circles: these inexpensive cards can easily be constructed or found in commercial materials as black-line masters. Students should colour and cut out their circles, using a common colour for each fraction (e.g., thirds – green, quarters – blue).</p> <p>Adding fractions with common denominators (same colour) is straightforward, but students will be challenged to describe their answer when adding $\frac{1}{4} + \frac{1}{3}$ (1 blue + 1 green). A common denominator (common colour) must be found to describe the sum.</p> <p>5. Writes number sentences to describe the addition and subtraction of fractions.</p>	<p>The intention here is for students to demonstrate their understanding of the operation. It is not necessary for students to solve the question but to be able to write down what is being demonstrated with the concrete manipulatives.</p> <p>This objective directly relates to the previous one and should be tied to the teaching of that objective.</p>	

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
C. Fractions 6. Uses concrete manipulatives to demonstrate the multiplication and division of proper fractions.	<p>Multiplication Illustrate this concept using a chocolate bar. e.g.,</p>  <p>Eight friends share this chocolate bar equally. Shade the portion each person receives.</p> <p>Bill decides to eat 1/2 of his share. Shade the portion he eats.</p> 	(R) Continue allowing students to use concrete manipulatives or pictorial form to develop full understanding.	<p>"Fraction Challenge II" MAC Courseware 7, Disk B.</p> <p><u>Problem</u></p> <p>A farmer died leaving 17 cows. According to the terms of his will the eldest child was to receive 1/2 of the cows, the second child to receive 1/3 and the youngest child to receive 1/9. The children were puzzled about how to carry out the terms of their father's will since none of these fractional parts of 17 cows was a whole number. Finally, a generous neighbour offered to loan a cow to the children. They then had 18 cows: 1/2 of 18 was 9 cows; 1/3 of 18 was 6 cows; and 1/9 of 18 was 2 cows. The total $9 + 6 + 2$ was the original 17 cows. The 18th cow remained to be returned. How was this possible?</p> <p>(See Journeys in Math 7 TRM, pp. 186-189.)</p>
	<p>Division</p> <p>Review the concept of division as repeated subtraction, then ask "how many times can 1/4 be subtracted from 1 1/2?"</p> 	<p>Students demonstrate their understanding by writing a number sentence.</p> <p>$1/2 \text{ of } 1/8 = 1/16$ (Discuss the meaning of the word "of".)</p> <p>Review the concept of division as repeated subtraction, then ask "how many times can 1/4 be subtracted from 1 1/2?"</p>	<p>Students demonstrate their understanding by writing a number sentence.</p> <p>(See Journeys in Math 7 TRM, pp. 186-189.)</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>C. Fractions</p> <p>7. Writes number sentences to describe the multiplication and division of fractions.</p>	<p>This objective should be taught with the previous objective. Students need only describe the concrete operations using number sentences, and should not be encouraged to compute at a formal level.</p>	<p>Remedial: Have students keep a temperature log for a week. Discuss daily changes; for example, Monday to Wednesday, or other variations.</p>	<p>Calculator Activity – Holtmath 7 Teacher's Edition, p. 257.</p>
<p>D. Integers</p> <ol style="list-style-type: none"> Maintains previously developed skills with integers (concept of integers, need for integers, ordering of integers). Uses concrete manipulatives to demonstrate the addition of integers. Writes number sentences to describe addition of integers. 	<p>Demonstrate the need for integers using:</p> <ul style="list-style-type: none"> change in temperature balancing chequebooks above and below sea level golf scores of above and below par time zones. 	<p>(Concrete Activities, Journeys in Math 7 TRM, pp. 292-303.)</p>	<p>(E) Journeys in Math 7 TRM, p. 299.</p> <p>MAC 7, Disk C.</p> <p>Holtmath 8, Disk 5.</p> <p>This objective should be combined with the previous objectives. Students should write number sentences to describe the concrete activity (adding integers). Adding integers at a formal level (development of addition rules) is discouraged.</p>

RATIO AND PROPORTION

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>1. Applies and practises problem-solving skills in new situations.</p> <p>2. Maintains previously developed skills (identifies ratios as ordered pairs of numbers related to concrete situations; uses whole number constants to generate equivalent ratios).</p> <p>3. Uses concrete manipulatives to construct ratios in the following forms: $a:b$, a as to b, and $\frac{a}{b}$</p>	<p>The intent of this objective is that every new situation (especially if it is perplexing), be approached from a problem-solving perspective. Every opportunity to teach a new problem-solving strategy should be taken when developing or reviewing concepts in this strand.</p> <p>Concrete Activities, Journeys in Math 7 TRM, pp. 204-207.</p> <p>Holtmath 7 Teacher's Edition, p. 191 "Alternative Teaching Strategies".</p>	<p>(R) Encourage the extended use of manipulatives for remediation.</p> <p>(E) Have students develop their own parallel concrete activity.</p> <p>The ratio $\frac{3}{6}$ can be expressed as 3:6 and 3 as to 6.</p> <p>Ensure that these forms are used interchangeably.</p> <p>When evaluating the objective, present ratios in all three forms.</p>	

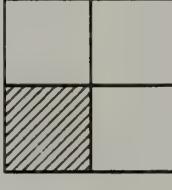
RATIO AND PROPORTION

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
4. Verifies the equivalence of two ratios using common multiples or factors: e.g., $\frac{14}{6} \rightarrow \frac{7}{3}$	Concrete Activities, Journeys in Math 7 TRM, p. 206 Holtmath 7 Teacher's Edition, p. 193 "Alternative Teaching Strategies". See Holtmath 7 Teacher's Edition, p. 193.	(R) Students maintain facility at a concrete level. (E & R) Journeys in Math 7 TRM, p. 211.	MAC 8 (9A) Ratio Rendezvous
5. Finds a missing element of a proportion using a common multiple of the elements: e.g., $\frac{3(x)}{4(x)} \rightarrow \frac{x}{12}$	The intent is to encourage the development of equivalence rules. (Do not allow the use of cross products.) Concrete Activities, Journeys in Math 7 TRM, p. 210.	(R) Students maintain facility at a concrete level (see #2).	

RATIO AND PROPORTION

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>6. Identifies percent as a ratio: e.g., $(p:100 \text{ or } \frac{p}{100})$</p>	<p>Use 10×10 cm grid paper. Have students calculate the total number of squares. Shade a portion and write the ratio of the shaded portion to the whole. Write the percentage of the shaded portion to the whole.</p> <p>Have students develop multiple examples to generate a rule for writing a percentage as a ratio.</p>	<p>(E) Have students estimate percents on test scores. Have them check on their calculators.</p> <p>(E) Explore the origin of the root word "cent"</p> <p>e.g., <u>century</u></p> <p>(R) Journeys in Math 7 TRM, pp. 236-237.</p>	 <p>10 cm</p> <p>e.g., Total 100 Shaded 25</p> <p>Ratio $\frac{25}{100}$ Percent 25%</p> <p>• $\frac{25}{100}$ is 25%</p>

RATIO AND PROPORTION

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>7. Expresses ratios as percents and decimals and vice versa (limit: ratios in the form a:b, where b = 2, 4, 5, 10, 20, 25, 50):</p> <p>e.g., $\frac{3}{4} \Rightarrow \frac{75}{100} \Rightarrow 75\%$</p>	<p>Emphasis should be placed on the notion that numbers have different equivalent forms.</p> <p>Concrete Activities, Journeys in Math 7 TRM, p. 240.</p> <p>The emphasis should be on simple ratios and mental conversions.</p> <p>Drill cards will develop ability to convert from one to the other.</p>	<p>(R) Journeys in Math 7 TRM, p. 241.</p> <p>(Journeys in Math 7 TRM Teaching Aids Game 12, p. 76.)</p>	<p>Students should use their calculators to find percents of their own test scores.</p>



Cover one vertex and have student verbalize what is missing. Extend this by covering two vertices.

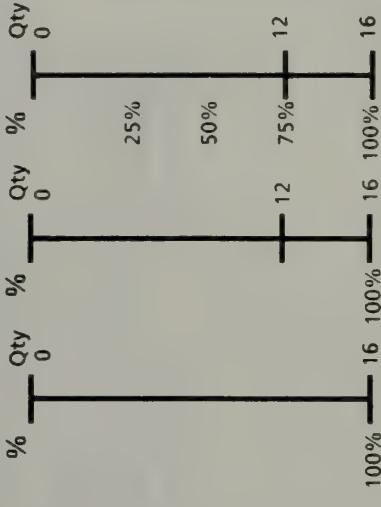
RATIO AND PROPORTION

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
8. Finds the percent of a number: e.g., 15% of 25	<p>To give meaning to this objective, some background preparation is necessary. Students should demonstrate their understanding of percent in concrete terms.</p> <p>e.g., Shade: 50% of a pie 100% of a pie 75% of a pie 25% of a pie 10% of a pie 1% of a pie</p> <p>e.g., Money 50% of \$1.00 = 50¢  25% of \$1.00 = </p>	<p>(E) Journeys in Math 7 TRM, p. 243.</p>	<p>Estimate percentages of numbers. Check using calculators.</p> <p>Develop utility program to find percentages of numbers.</p>

RATIO AND PROPORTION

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>9. Expresses one number as a percent of another number:</p> <p>e.g., 12 is what percent of 16?</p> <p>or $\frac{12}{16} = \frac{\text{ }}{\text{ }}$</p>	<p>Discuss in terms of real life examples. Where would the percent of a number be used?</p> <p>e.g., test scores sports results</p>	<p>(E) The sale price of an article is \$12. If the original cost was \$16, what percentage was saved during the sale?</p> <p>Emphasis can be placed on <u>estimation</u> and the use of the calculator/computer to check the result.</p> <p>e.g., 12 is what % of 16?</p> 	<p>Fig. I</p> <p>Fig. II</p> <p>Fig. III</p> <p>- Place 16 opposite 100% - Estimate and position 12 - Subdivide scale as necessary (50%, 25%)</p>

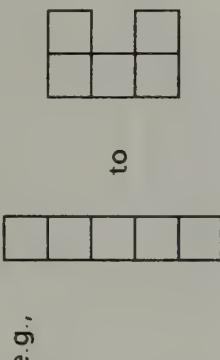
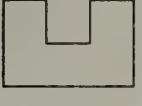
MEASUREMENT AND GEOMETRY

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>1. Applies and practises problem-solving skills in new situations.</p> <p>2. Maintains previously developed skills.</p>	<p>Introduce concept of measurement. Use examples such as labels on food packages, speedometer numbers, sporting event distances.</p>	<p>Measurement should be taught as "a comparison to some arbitrary unit". These units are repeated and may be combined into larger units. The need to subdivide a unit should be demonstrated when a fraction of a unit is needed to describe a length, mass, etc. Initially the arbitrary unit may be one's hand. The need to standardize units (metres) should evolve.</p> <p>a) linear</p>	<p>Problem</p> <p>Using maps and distance charts to solve problems. Students will need to interpret maps and charts, and plan using more than one possible path to a solution.</p> <p>When measuring they can express the units (in m and cm).</p>

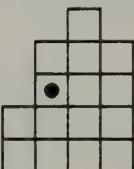
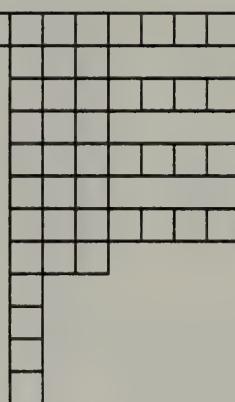
MEASUREMENT AND GEOMETRY

Grade 7

Objective	Clarification or Example	Elective Suggestions
<p>2. Maintains previously developed skills. (cont'd)</p> <p>b) perimeter</p>	<p>Estimate and measure the distance around various rectangular and triangular objects.</p> <p>c) area</p> <p>Using a ruled graph paper, cut out rectangles and triangles then determine their area by counting squares. Have students develop a strategy (such as a formula) for determining the area of rectangles or triangles.</p> <p>(E) Explore how rearranging 5 square cards changes the perimeter.</p> <p>e.g.,</p>  <p>(E) Computer: Mathematics 7 Activities Courseware, Metric Mysteries, Houghton Mifflin.</p> <p>Remedial</p> <p>*Create a Creature:</p> <p>Designate an area, such as 60 cm^2. Have the children cut out the area of squared paper into as many sections as they wish and then arrange the sections to form a creature. They will also enjoy creating creatures with the areas of certain parts specified. For example, they might use 24 cm^2 for the body, 16 cm^2 for the legs, and 20 cm^2 for other parts. Such a fanciful creature is shown on the next page.</p>	<p>Problem</p> <ul style="list-style-type: none"> Explore perimeters of compound figures. <p>e.g.,</p>  <ul style="list-style-type: none"> Using a variety of tools such as string, grid paper, ruler, etc., have students try to determine the area of classroom objects such as a book front, desk top. A square decimetre is a handy size for comparing with other objects. Have each child cut a square decimetre ($10\text{ cm} \times 10\text{ cm}$) from squared paper. Specify several objects and ask them to compare the area of each to the square decimetre. They might use objects such as their textbook covers, a single surface of their crayon boxes, the area of their hands (outlined on paper), or the area of their desk tops. Lead them in a discussion of strategies for estimating area.

MEASUREMENT AND GEOMETRY

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>2. Maintains previously developed skills. (cont'd)</p> <p>c) area</p>	<p>A final project might be making creatures of any size and then calculating the area of the finished cutouts.</p> <p></p> <p></p>	<p>They might lay their square decimetres over an object whose area they want to estimate to see if the object's area is greater than, less than, or about equal to that of the square decimetre. They might see approximately how many times the decimetre square can be placed on a large area. They might also lay a smaller object on top of the square decimetre and estimate the fraction of the square decimetre that is taken up by the object.</p>	<p><i>* (Arithmetic Teacher, Vol. 31, #4, December 1983, pp. 4-11.)</i></p>

MEASUREMENT AND GEOMETRY

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>2. Maintains previously developed skills. (cont'd)</p> <p>c) area</p>		<p>*Relating perimeter and area</p> <p>Instruct children to cut out as many rectangles and other polygons with a given area as they can. They should then figure the perimeter of each polygon and put the polygons in order from smallest to largest in perimeters. The class should share their results and list as many solutions to the problem as possible.</p> <p>Next, have students cut out four rectangles, each having an area of 48 cm^2. They will realize that the shapes of these can vary. After they have cut out their four rectangles, invite them to cut each of them into two or more pieces to form another polygon. They can rearrange the pieces to form L-shaped figures (hexagons) and other polygons. Ask them to find and label the perimeter of each polygon. One-centimetre segments can be counted to find the perimeter for some polygons; for others in which the squared paper has not been cut at right angles, rulers can be used to find the lengths of the sides, or the perimeter of the polygons can be estimated.</p>	<p>*(Arithmetic Teacher, Vol. 31, #4, December 1983, p. 11.)</p>

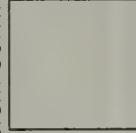
MEASUREMENT AND GEOMETRY

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>2. Maintains previously developed skills. (cont'd)</p> <p>d) volume</p>	<p>Have students construct rectangular prisms with centimetre cubes, and develop a strategy for determining the volume. Extend this to practical situations such as the volume of an aquarium.</p>	<p>In general, capacity units are used for liquids. The relationship of $1 \text{ mL} = 1 \text{ cm}^3$ can be explored by using a waterproof rectangular solid, comparing the water it can hold to the cubic centimetre blocks it can hold.</p> <p>Explore the use of capacity in practical situations such as the capacity of a fuel tank, a pop bottle, a swimming pool, a grain bin.</p>	<p><u>Problem</u></p> <p>Have students explore what happens to the volume of a cube if the length of a side is doubled. Use a calculator, prepare a chart.</p> <p>e) capacity</p> <p>f) mass</p> <p>The mass of an object is "the amount" of material in that object. Two common mass units are the kilogram and gram. The feel of these can be found by estimating masses of objects by picking them up and then finding their actual mass on a balance. The appropriateness of other SI units (milligram, tonne) should be explored.</p>

MEASUREMENT AND GEOMETRY

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>2. Maintains previously developed skills. (cont'd)</p> <p>g) angle</p>	<p>Estimate angles, and then measure using a protractor. Some concrete examples are the hands of a clock, the corner on a table, the blades of scissors.</p> <p>Discuss the inside and outside scale on the protractor.</p>	<p>(E) Explore tiling patterns (tessellations). Using shapes, try to tile a plane or design a tessellation with one or more shapes.</p> <p>(Arithmetic Teacher, Vol. 31, #5, January 1984, p. 54.)</p>	<p><u>Problem</u> How do you measure an angle whose rays are too small, or the angle that is greater than 180°?</p> <p>Computer-Mathematics Activities Courseware 7, Houghton Mifflin, Congruent Figures, p. 63.</p> <p>Problem-Solving Activity: Draw 2 lines so the picture shows 4 congruent triangles. </p> <p>(Arithmetic Teacher, Vol. 31, #5, January 1984, p. 31.)</p>
<p>h) transformational geometry</p>	<p>Identifies and draws translations (slides), reflections (flips) and rotations (turns) of figures, and tests for congruence.</p>	<p>(E) Explore through concrete situations the relationship of SI units of area, volume, and mass.</p> <p>(R) Arrange students in groups. One group will draw lines of, for example, 5.2 cm, 6.5 cm, 7.8 cm, 15.0 cm. Another group will make line segments of 52 mm, 65 mm, 78 mm, 150 mm. Groups then try to find a match.</p>	
<p>3. Expresses equivalent measures of SI units (linear).</p>	<p>The appropriateness of units should be explored. For example, the distance from the school to home should be measured in metres or kilometres but not centimetres. Conversions used in practical situations should be developed.</p>		

MEASUREMENT AND GEOMETRY

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
4. Understands and uses the terms similar and congruent with respect to geometric figures.	<p>Congruence is having the same size and shape. Similar is having the same shape but not necessarily the same size (apply transformational geometry).</p> <p>Students can be encouraged to draw congruent and similar shapes.</p> <p>Journeys in Math 7 TRM, pp. 355, 378-379. Holtmath 7 Teacher's Edition, pp. 332-333.</p>	<p>Explore lines of symmetry by dividing a given shape into two congruent parts by folding or by using a mira. Using a mira, draw the reflection in the mira and make a new shape with a line of symmetry. This can be reinforced by transformational geometry (flips). The lid of a square box can be turned four different ways to fit the box. It has turn symmetry. The order of turn symmetry for a figure is the number of times it fits onto itself in one full turn. The order of turn symmetry for the square box is 4. Have students explore turn symmetry by tracing shapes and turning them to find their order of turn symmetry.</p> <p>(E) Discuss graphic design by M.C. Escher. Produce patterns with turn symmetry.</p>	
5. Understands and uses the terms symmetry with respect to geometric shapes.			

MEASUREMENT AND GEOMETRY

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>5. Understands and uses the term symmetry with respect to geometric shapes. (cont'd)</p> <p>6. Constructs geometric designs using tools such as a computer, compass, straightedge, ruler or mira.</p>	<p>Lines of symmetry and turn symmetry can be reinforced by examining trademark designs of companies such as Shell Oil, Westinghouse Electric, Ralston Purina, Chrysler Corporation, or Mattel. A strategy to develop students' interest is to create an attractive bulletin board displaying examples of trademarks with line symmetry, turn symmetry, as well as combinations. Some non-examples might also be included.</p> <p>Some possible designs are company logos, tessellations of the plane (M. C. Escher). By exploring various designs, students learn the use of geometric tools and explore properties of geometric figures.</p> <p>Encourage students to be creative. One way is to arrange colourful and attractive bulletin board displays of geometric designs.</p> <p>(See Journeys in Math 7 TRM, p. 149.)</p>	<p>A LOGO computer program may be used to create designs.</p> <p>Journeys in Math 7 TRM, pp. 117, 120.</p>	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>1. Applies and practises problem-solving skills in new situations.</p>	<p>The problem-solving framework should be re-emphasized: understanding the problem, developing a plan, carrying out the plan, and looking back.</p> <p>Problems should not be limited to traditional word problems, but should include situations from the student's environment as much as possible.</p>		<p>Objectives can be designed to incorporate problem solving into data management, such as:</p> <ul style="list-style-type: none"> a) The student would ask questions in which statistical data would help to find an answer. b) The student would decide what type of statistical measure would help to find the answer. c) The student would collect and compute the necessary data. d) The student would organize and interpret the data. e) The student would answer the question. f) The student would recognize the other choices of data and other interpretations, and should therefore be ready to defend or modify the conclusions.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Demonstrates a knowledge and understanding of the use and purposes of statistics as it affects daily living.	<p>The students can use newspapers to locate information such as weather reports, prices of used cars, sports statistics, and fashion prices. They should then display this information and find the high price, low price and "typical" price. The students should be able to answer the following questions:</p> <p>a) How do numbers affect our daily lives? b) What do the numbers compare? c) List the possible sources of data: e.g., newspapers, magazines, books, radio, television, personal experiences, sports cards, opinion polls, local surveys, etc. d) Why are statistics kept? e) How do statistics help us? f) How are statistics displayed clearly?</p> <p>By relating to the students' environment, discuss how numbers are used in their daily lives. Use the above sources to explain what is being compared, why statistics are kept, how they help us, and how the results can be displayed (tables, graphs, etc).</p>	<p>(E) Investigate what a statistician is, and report what a career as a statistician would be like.</p>	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3. Collects and records data (tally sheets and frequency tables).	<p>Use questions which require the students' output such as:</p> <p>a) How many kilometres (or blocks) is your home from school?</p> <p>b) What is your hair colour?</p> <p>c) What is your eye colour?</p> <p>d) What type of shoes are you wearing now?</p> <p>e) What is your favourite sport?</p> <p>f) Research data in the school library.</p> <p>Use the tally method of fives () to record the number of occurrences, and then write down the frequency counts.</p> <p>The students answer questions concerning the greatest, smallest, and most typical occurrences. Use a month's temperature data for a town or city to find a way to organize the high and/or low temperatures into frequency tables, means, and ranges.</p>	<p>(E) Using data from newspapers have the student make frequency tables, make up five questions on the data, exchange the data and questions with another student, and answer each other's questions.</p> <p>(E) Use 49 cards (labelled 1 to 49) and play "Lotto 6-49" 20 times (draw 6 numbers each time), and record the results.</p> <p>(Journeys in Math 7 TRM, p. 321.)</p>	

DATA MANAGEMENT

Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>4. Understands and uses the term average (mean) as related to practical situations (e.g., test marks).</p>	<p>Use information such as bowling scores, test scores, students' weights, etc. The students will determine averages (means) by adding all the items (maximum of three digits per number) and dividing by the number of items.</p> <p>Have groups of students list and compare items (e.g., number of hours of TV watching, ages in months, etc.).</p> <p>(Journeys in Math 7 TRM, p. 336.)</p>	<p>(R) Use smaller numbers and restrict sample size to determine mean.</p> <p>(E) Find the missing mark in a list of marks when the mean is given.</p>	<p>Write a computer program that will calculate the mean for any list of numbers.</p> <p>Houghton Mifflin MAC 8, The Three M's.</p> <p>Computer Average Program, Holtmath 7 Teacher's Edition, p. 301.</p>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
5. Interprets data from pictographs, bar graphs, line graphs and circle graphs.	<p>A graph displays and compares information. A pictograph uses symbols to display the data conveniently. A bar graph uses intervals and scales to display data. A line graph displays the relationship between two changing quantities. A circle graph represents data as being part of a whole.</p> <p>Using graphs from any available sources, the students should be able to identify the types of graphs and to read information from each graph.</p>	<p><u>Pictographs</u></p> <p>(R) Practise skip counting by 25, 10 or whatever unit used in the pictograph.</p> <p><u>Bar graphs and line graphs</u></p> <p>(R) Use a square corner of plastic to align the horizontal and vertical scales, and check to see that students are reading the scales correctly.</p> <p><u>Circle graphs</u></p> <p>(R) Review measuring angles in combination with fraction circles.</p>	<p>Houghton Mifflin MAC 7, Pie Graphics.</p> <p>Use LOGO to construct graphs.</p>
6. Understands when and how to represent data in the form of pictographs, bar graphs, line graphs and circle graphs.			<p>Each type of graph has its strengths and weaknesses. Pictographs are easy to read and appealing to look at but are not always very accurate. A bar graph is more accurate but is less appealing to look at. A line graph is best for displaying the relationship between two changing quantities, but has limits to its accuracy and care must be taken in making predictions based on trends.</p> <p><u>Pictographs</u></p> <p>(E) Make pictographs that compare large numbers.</p> <p>(R) Use topics which lend themselves to pictorial representation.</p>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
6. Understands when and how to represent data in the form of pictographs, bar graphs, line graphs and circle graphs. (cont'd)	<p>A circle graph is good for displaying data as parts of the whole but can be more difficult to compare with other graphs.</p> <p>The students can use data sources (such as newspapers, surveys or from previous exercises) to draw various types of graphs. The students should use knowledge of the strengths and weaknesses of each type of graph to select the form in which the data should be represented.</p> <p>The students should be introduced to the steps in drawing a circle graph:</p> <ol style="list-style-type: none"> find the angle measure by multiplying each percent times 360°. construct a circle and use a protractor to draw each angle label the graph. 	<p><u>Bar graphs</u></p> <p>(E) Discuss when to use horizontal bar graphs and when to use vertical bar graphs.</p> <p>(E) Use scales that vary (don't start at 0).</p> <p><u>Line graphs</u></p> <p>(R) Use a line graph from a source to write a report describing what the graph is about and then reproduce the graph.</p> <p>(E) Draw a double line graph (e.g., high/low temperatures, populations of Calgary and Edmonton over a period of years).</p> <p><u>Circle graphs</u></p> <p>(R) Review how to find percents of numbers.</p> <p>(E) Given a circle graph with unlabelled sectors, have the students calculate what percent each sector represents.</p>	

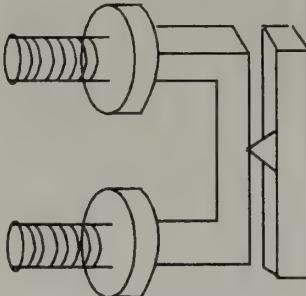
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>1. Applies and practises problem-solving skills in new situations.</p> <p>2. Understands and uses the term variable and uses variables to describe a concrete situation (e.g., number of jelly beans in a jar).</p> <p>Use concrete situations to demonstrate the meaning and use of variables (a letter or symbol that represents an unknown number). For example, a variable can be used to express the number of pennies in a jar, or the age of a teacher.</p>	<p>Specific problem-solving techniques and skills will be developed with each objective.</p> <p>Many BASIC and LOGO programs use variables in input statements. By varying the input value, the students can explore the effects. For example:</p> <p>LOGO Program</p> <pre>TO SQUARE : SIDE FD : SIDE RT 90 END</pre> <p>SQUARE 30 draws a square with side length of 30, SQUARE 40 draws a square with side length 40, and so on.</p>		

ALGEBRA**Grade 7**

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3. Uses variables to write mathematical expressions to represent practical situations (e.g., age of the students in the class in three years will be $x + 3$ years).	<p>With the inclusion of the operations $+$, $-$, \times and \div, we can represent practical situations using open mathematical expressions. For example, when three pennies are added to a jar containing an unknown number of pennies, we can express this by $x + 3$.</p> <p>Use similar procedure for other simple expressions:</p> <p>three less than a number: $x - 3$ 5 times my age: $5 \times x$ or $5x$.</p> <p>Extend the activity to more complex statements: three more than 5 times a number: $5x + 3$. Give students an expression such as $3a - 2$ and have them state its meaning in words.</p>		

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology																						
<p>4. Evaluates expressions for given values of the variable (limit: whole numbers, positive rationals).</p>	<p>Expressions may be evaluated mentally, formally by substitution or organized in the form of tables.</p> <p>Use practical situations to reinforce the use of mathematical expressions. For example, movie admission is \$2.50. A table is one method of showing costs for different numbers of people.</p> <table data-bbox="658 1183 894 1500"> <thead> <tr> <th data-bbox="658 1183 706 1500">Number</th> <th data-bbox="658 1183 706 1500">Cost</th> </tr> </thead> <tbody> <tr> <td data-bbox="706 1183 753 1500">1</td> <td data-bbox="706 1183 753 1500">\$2.50</td> </tr> <tr> <td data-bbox="753 1183 800 1500">2</td> <td data-bbox="753 1183 800 1500">\$5.00</td> </tr> <tr> <td data-bbox="800 1183 847 1500">3</td> <td data-bbox="800 1183 847 1500">\$7.50</td> </tr> <tr> <td data-bbox="847 1183 894 1500">p</td> <td data-bbox="847 1183 894 1500">\$2.50 × p</td> </tr> </tbody> </table> <p>The expression for cost is $\\$2.50 \times p$ or $\\$2.50p$.</p>	Number	Cost	1	\$2.50	2	\$5.00	3	\$7.50	p	\$2.50 × p	<p>Looking for a pattern is one problem-solving technique.</p> <p><u>Problem</u></p> <p>There are 18 animals in the barnyard. Some are chickens, some are cows. There are 50 legs in all. How many are chickens and how many are cows? By making a table, and examining data, a pattern emerges.</p> <table data-bbox="753 169 862 549"> <thead> <tr> <th data-bbox="753 169 800 549">Chickens</th> <th data-bbox="800 169 847 549">17</th> <th data-bbox="847 169 894 549">16</th> <th data-bbox="894 169 941 549">15</th> </tr> <tr> <th data-bbox="800 549 847 549">Cows</th> <th data-bbox="847 549 894 549">1</th> <th data-bbox="894 549 941 549">2</th> <th data-bbox="941 549 988 549">3</th> </tr> <tr> <th data-bbox="894 549 941 549">Legs</th> <th data-bbox="941 549 988 549">38</th> <th data-bbox="988 549 1035 549">40</th> <th data-bbox="1035 549 1082 549">42</th> </tr> </thead> </table> <p>Use a calculator's automatic constant feature for \times, \div, $+$ and $-$. For each operation have students determine which operation is the automatic constant for their own calculator.</p>	Chickens	17	16	15	Cows	1	2	3	Legs	38	40	42	
Number	Cost																								
1	\$2.50																								
2	\$5.00																								
3	\$7.50																								
p	\$2.50 × p																								
Chickens	17	16	15																						
Cows	1	2	3																						
Legs	38	40	42																						

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology								
<p>4. Evaluates expressions for given values of the variable (limit: whole numbers, positive rationals). (cont'd)</p> <p>Extended Content</p> <p>Find the mean from a frequency table.</p>	<p>From a table of values, students try to determine the defining expression by looking at the pattern.</p> <table border="1" data-bbox="478 1469 651 1553"> <tr> <td>x</td> <td></td> </tr> <tr> <td>1</td> <td>3</td> </tr> <tr> <td>2</td> <td>4</td> </tr> <tr> <td>3</td> <td>5</td> </tr> </table> <p>The defining expression is $x + 2$.</p>	x		1	3	2	4	3	5	<p>With the basic operations ($+$, $-$, \times, \div) and the inclusion of $=$, we can represent practical situations using closed mathematical sentences.</p> <p>Mathematical sentences (equations) may have one unknown (i.e., a number plus 3 equals 7 becomes $x + 3 = 7$) or have more than one unknown (i.e., students in the room = boys + girls becomes $s = b + g$).</p> <p>5. Uses variables to write mathematical sentences to represent practical situations (e.g., people in a classroom = boys + girls + teachers or $p = b + g + t$).</p>	<p>Houghton Mifflin MAC 7.</p> <p>Problem</p> <p>For the normal range of summer temperatures the number of chirps made by a cricket in a minute is predictable. For example, at 16°C the cricket chirps 140 times per minute. At 24°C the cricket chirps 172 times per minute. What is the temperature at which the cricket chirps 180 times per minute? At 21°C, how many times will the cricket chirp per minute?</p>
x											
1	3										
2	4										
3	5										

Objective	Clarification or Example	Elective Suggestions Integration of Problem Solving and Technology
<p>6. Uses concrete manipulatives to demonstrate the concept of "equals" (i.e., equality).</p>	<p>Using a balance scale and one gram masses, students should experiment to discover balance by manipulating the placement of the one gram masses.</p> <p>Having discovered that an equal number of one gram masses is required on each side of the balance, students should replace the one gram masses on one side of the balance with different size masses but maintaining balance.</p>	<p><u>Problem</u></p> <p>You have a pile of 24 coins. Twenty-three of these coins have the same weight and one is heavier than the others. Your task is to determine which coin is heavier. You are given a balance beam which will compare weights. Develop a strategy to find the heavier coin in a minimum number of weighings.</p>  <p>$19 \quad 19 \quad 19$</p> <p>$19 \quad 19 \quad 19 \quad 19$</p> <p>$19 \quad 19 \quad 19 \quad 19 \quad 19$</p> <p>$19 \quad 19 \quad 19$</p> <p>$29 \quad 59$</p> <p>$7 = 7$</p> <p>$2 + 5 = 7$</p> <p>Thus students should discuss the concept of equality as a state of balance. For example:</p> <p>$7 = 7$</p> <p>$2 + 5 = 7$</p> <p>$3 + 4 = 7$</p> <p>$1 + 6 = 7$</p>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology																				
6. Uses concrete manipulatives to demonstrate the concept of "equals" (i.e., equality). (cont'd)	<p>The forms used are limited to one unknown using whole numbers, positive fractions and positive decimals. Considerable time should be spent on the concept that an equation is a balance in which the left side equals the right side. A possible manipulative is a balance scale.</p>	<p>(E) A simple classroom game is to have a student pick a secret number. Ask the student to perform some operation on that number (e.g., add 2) and then tell the class the result. Have class members find the secret number. This can be expanded to writing the equation and checking the solution. The game can be expanded to include more than one operation and the sequence of the operations.</p>	<p>Use of calculators as a means of guess and test.</p> <p>The automatic constant function is a valuable aid to guess and test.</p> <p>e.g., To solve $\frac{x}{5} = 4.2$</p> <table style="margin-left: 200px;"> <tr> <td>$x = 10$</td> <td>$10 \div 5 =$</td> <td>2</td> <td>too low</td> </tr> <tr> <td>$x = 15$</td> <td>$15 \div 5 =$</td> <td>3</td> <td>too low</td> </tr> <tr> <td>$x = 20$</td> <td>$20 \div 5 =$</td> <td>4</td> <td>too low</td> </tr> <tr> <td>$x = 23$</td> <td>$23 \div 5 =$</td> <td>4.6</td> <td>too large</td> </tr> <tr> <td>$x = 21$</td> <td>$21 \div 5 =$</td> <td>4.2</td> <td>✓</td> </tr> </table> <p>(R) "Roll It". Prepare cards with equations having whole number solutions between 2 and 12. For example,</p> <div style="border: 1px solid black; padding: 5px; display: inline-block;">$x + 1 = 7$</div> <div style="border: 1px solid black; padding: 5px; display: inline-block;">$5b = 15$</div>	$x = 10$	$10 \div 5 =$	2	too low	$x = 15$	$15 \div 5 =$	3	too low	$x = 20$	$20 \div 5 =$	4	too low	$x = 23$	$23 \div 5 =$	4.6	too large	$x = 21$	$21 \div 5 =$	4.2	✓
$x = 10$	$10 \div 5 =$	2	too low																				
$x = 15$	$15 \div 5 =$	3	too low																				
$x = 20$	$20 \div 5 =$	4	too low																				
$x = 23$	$23 \div 5 =$	4.6	too large																				
$x = 21$	$21 \div 5 =$	4.2	✓																				
7. Uses estimation, and guess and test procedures to solve equations of the form: $x + a = b$, $ax = b$, $ax + b = c$, and $\frac{x}{a} = \frac{b}{c}$	<p>Solve $2x + 3 = 15$ by guess and test</p> <p>Try $x = 5$ $2 \times (5) + 3 = 13$ too small Try $x = 6$ $2 \times (6) + 3 = 15$ ✓</p> <p>This reinforces substitution of expressions and organizing work to document the process.</p>																						

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
7. Uses estimation, and guess and test procedures to solve equations of the form: $x + a = b$, $ax = b$, $ax + b = c$, and $\frac{x}{a} = \frac{b}{c}$ (cont'd)	Deal 5 cards to each player. The first player rolls a pair of dice. Each player who has a card with an equation whose solution matches the dice discards that card. A discard may be challenged. If the discard was not correct, the player must pick up the discard and also take one from the challenger. The first player with no cards wins.	(E) Journeys in Math 7 TRM, p. 277.	Encourage the use of a calculator. This reinforces the order of operations.
8. Verifies solutions to equations by substitution.	Verification of a solution is simply trying to determine if the left side balances the right side.	<p>Is 6 the solution to $2x + 3 = 13$?</p> $\begin{aligned} 2x() + 3 &= 13? \\ 2x(6) + 3 &= 13? \\ 12 + 3 &= 13? \\ \bullet\bullet 15 &\neq 13 \end{aligned}$	(Journeys in Math 7 TRM, pp. 276-277.)

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
9. Given ordered pairs, plot points on a coordinate plane.	<p>This topic should be familiar to students and so need not be dealt with at great length. An interesting activity is to plot points in all 4 quadrants that determine a picture or design.</p>	<p>(E) Give students simple pictures or designs and have them transcribe it to a coordinate plane then to identify the points needed to reproduce their picture.</p> <p>Have students exchange with a classmate for checking by reproducing the picture from the given points.</p>	Holtmath 7 Teacher's Edition, pp. 314-315. Software Disk 6, Graphing Ordered Pairs.

(

(
66)

(

GRADE 8
MATHEMATICS

(

69

(

PLEASE NOTE

The following resources have received basic status and are available from the Learning Resources Distributing Centre:

Student texts for *JOURNEYS IN MATH 7, 8*
Student texts for *HOLTMATH 7, 8, 9*

Any other titles mentioned in this guide have not received formal status approval. They are presently under review and cannot, at this time, be obtained through the Learning Resources Distributing Centre.

CURRICULUM GUIDE (DRAFT)

PROBLEM SOLVING

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Demonstrates an understanding of a problem-solving situation.	<p>Problem solving should not be viewed as an isolated activity but rather as a process that is to be an integral part of the teaching philosophy to be used in the development of the other strands. The framework for problem solving should be introduced at the beginning of the year (suggested time – 3 to 5 periods).</p> <p>Brainstorm for a definition and examples. The following ideas should evolve about a problem:</p> <ul style="list-style-type: none"> a) it has no readily apparent solution or the means to the solution is not immediately evident b) it can cause a person to be temporarily perplexed c) it may have no answer, one answer, or more than one answer <p>Teachers must recognize that problem-solving skills are essential for all students and that being perplexed when first encountering a problem is normal. Problems presented to students should be challenging yet solutions must be attainable to insure that students experience success.</p> <p>It is very important for teachers to realize individual student differences in learning; therefore the growth expectation should also vary.</p> <p>Individual needs can often be met by changing the conditions of a problem to make it simpler.</p>	<p>Teachers must recognize that problem-solving skills are essential for all students and that being perplexed when first encountering a problem is normal. Problems presented to students should be challenging yet solutions must be attainable to insure that students experience success.</p> <p>It is very important for teachers to realize individual student differences in learning; therefore the growth expectation should also vary.</p> <p>Individual needs can often be met by changing the conditions of a problem to make it simpler.</p>	<p>a) The use of calculators in problem solving must be encouraged so that time spent on tedious calculations is decreased and feedback on strategies is faster. Numbers from realistic and relevant situations are less imposing if calculators are used.</p> <p>b) Group work should often be used in problem solving. A student in a group deals with ideas and questions from other members of the group, and this may help each student to progress in developing problem-solving strategies.</p>

Note: (E) = Enrichment
(R) = Remediation

PROBLEM SOLVING

Grade 8

Objective	Clarification or Example	Elective Suggestions Integration of Problem Solving and Technology
<p>1. Demonstrates an understanding of a problem-solving situation. (cont'd)</p> <p>d) it can be of a practical, everyday, personal or social nature as well as of a mathematical nature.</p> <p>(See Journeys in Math 7 TRM, p. 30.)</p>	<p>Manipulatives can also be used to meet individual needs.</p>	<p>e.g., Students are given a pile of 21 markers. Two players are involved and take turns removing one, two or three markers. The winner is the player who removes the last marker. The purpose of the game is to develop a strategy to always win. As students continue to work on this they should become more interested in finding a strategy rather than winning. For students who have difficulty with this, decrease the number of markers used or only move one or two markers. Demonstrate how the markers can be grouped and ask students critical questions such as the importance of moving first, and other strategic moves. The game can also be made into a more difficult version to challenge higher ability students. Use two piles and change the rules. Students can take one marker from each pile or one marker from only one pile.</p>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>2. Demonstrates a willingness to find a solution to a problem.</p> <p>In order to develop the students' willingness to find solutions, the teacher should:</p> <ol style="list-style-type: none"> create a positive classroom atmosphere that allows students to foster their own ideas and approaches in problem solving be supportive and encourage risk taking in finding solutions encourage students to use creative approaches be willing to accept unconventional solutions, more than one solution, or no solution (where appropriate) challenge students to think critically and justify strategies and solutions be enthusiastic and capable of recognizing the students' willingness and perseverance to solve problems provide appropriate questions for students present problem situations that enable students to gain problem-solving experience that is transferable to other subject areas and everyday life. 	<p>Students who experience difficulty with the complex strategies may find it necessary to use a more concrete approach for a longer period of time and may require more teacher guidance.</p> <p>e.g., A store owner buys candies in bulk bags containing 80 candies each. He re-packages the candies for sale in smaller bags of 12. How many candies are left over when one bulk bag of 80 candies is re-packaged?</p>		<p>Computers may be used to assist in teaching problem solving. Various programs and simulations require the use of particular or various strategies (e.g., Houghton Mifflin MAC, MECC, and Sunburst Communications Software).</p> <p>The use of relevant and realistic problems (from sources such as newspapers and magazines) is encouraged because this will increase the interest of students. Students may also be able to contribute their own ideas of problems to solve.</p>

PROBLEM SOLVING

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Demonstrates a willingness to find a solution to a problem. (cont'd)	<p>Use of a concrete example will help students who experience difficulty with the operation of division.</p> <p>Concrete approaches should be encouraged as long as it is necessary.</p> <p>A teacher should challenge the more capable students by having them not only justify their strategies and solutions but also to consider the possibilities such as:</p> <ul style="list-style-type: none"> a) other strategies and solutions b) "what if?" (change an element of the problem) c) generalization of rules to other situations. <p>e.g., Using the above candy problem, ask: "How many bulk bags of 80 candies each would the store owner need to re-package so that <u>no candies are left over</u>?"</p>		

PROBLEM SOLVING

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>3. Uses a variety of strategies to solve problems. Previously developed strategies are used.</p> <p>The following strategies should be developed throughout the various strands of the program and within the problem-solving framework:</p> <ul style="list-style-type: none"> a) Understanding the problem ● interprets pictures, charts and graphs ● asks relevant questions 	<p>Students should encounter new situations that require an extension of problem-solving strategies acquired in Grade 7. The approach to this may be similar to that employed at the Grade 7 level whereby three non-related but similar problems can be chosen to focus on a particular strategy.</p> <p>As a teacher demonstration, involving the class, the strategy of working backwards can be developed within the problem-solving framework (Understanding the problem, Developing a plan, Carrying out the plan, Looking back) as follows:</p> <p>The number of a past year is divided by 2 and the result turned upside down and divided by 3, then left right side up and divided by 2. Then the digits in the result are reversed to make 13. What is the past year? (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, p. 42, #28.)</p>	<p>The Scope and Sequence in the Teacher's Edition (Holtmath 8) and Teacher Resource Manual (Journeys in Math 8) identify specific problem-solving strategies.</p> <p>Under the guidance of the teacher, students investigate the strategy by solving a similar but non-related problem such as:</p> <p>A boy attempts to climb a 10 m pole. At every attempt he climbs 1 m and slips back $\frac{1}{2}$ m. After how many attempts will he have reached the top? (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, p. 29, #8, 12.)</p>	

PROBLEM SOLVING

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>3. a) Understanding the problem (cont'd)</p> <ul style="list-style-type: none"> ● interprets pictures, charts and graphs ● asks relevant questions 	<p>The natural progression leads to students using the same strategy to solve a problem such as:</p> <p>Janice went to a store, spent half of her money, and then spent \$10 more. She went to a second store, spent half of her remaining money, and then spent \$10 more. Then she had no money left. How much money did she have in the beginning when she went to the first store? (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, p. 10, #2.)</p>	<p>A teacher should challenge the more capable students by having them not only justify their strategies and solutions but also to consider the possibilities such as:</p> <ul style="list-style-type: none"> ● other strategies and solutions ● "what if?" (change an element of the problem) ● generalization of rules to other situations. <p><u>Evaluation</u></p> <p>The evaluation of problem solving requires more than grading the solutions to mathematical problems. Continual observation and questioning of students while they are solving problems is essential.</p> <p>In assessing a student's problem-solving skills, the teacher should consider:</p> <ul style="list-style-type: none"> ● willingness to attempt problems ● use of systematic approach ● selection of appropriate strategies ● efficiency in selection of appropriate strategies 	<p>Holtmath 8 Teacher's Edition, p. 83.</p>

PROBLEM SOLVING

Grade 8

Objective	Clarification or Example	Elective Suggestions Integration of Problem Solving and Technology
<p>3. b) Developing a plan (choosing a strategy)</p> <ul style="list-style-type: none"> ● collects and organizes information (charts and graphs) ● makes diagrams and models ● experiments through the use of manipulatives ● breaks the problem into smaller parts ● works backwards <p>c) Carrying out the plan</p> <ul style="list-style-type: none"> ● applies selected strategies ● presents ideas clearly ● documents the process ● works with care ● works in a group situation <p>d) Looking back</p> <ul style="list-style-type: none"> ● makes and solves similar problems. 	<ul style="list-style-type: none"> ● logical justification of strategies and solutions ● perseverance ● growth of confidence in problem-solving ability ● transfer of problem-solving skills to situations other than mathematics. <p>Evaluation techniques and instruments for problem solving are found in the Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, pp. 7, 8, 52-56.</p> <p>(See Holtmath 8 Teacher's Edition, p. iv Problem-Solving Strategies.)</p>	<p>e.g., Ann, Beth, Cathy, Dee, and Evita were playing in matches to see who would play position 1, 2, 3, 4, or 5 on the girls' tennis team. Each girl played each of the other girls once. How many matches were played?</p> <p>e.g., Children often use the <i>make-an-organized-list</i> strategy when solving this problem. First, a child might list all the matches that person A plays. Then the child would list all the matches played by persons B, C, D, and E. Note that once match AB is listed, match BA involves the same players and is not a different match.</p> <p>AB AC AD AE BC BD BE CD CE DE</p> <p>(Arithmetic Teacher, Vol. 32, #4, December 1984, p. 30.)</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology																			
<p>1. Applies and practises problem-solving skills in new situations.</p>	<p>The intent is to develop non-routine problem-solving skills in every student. Each student should understand the concepts of the problem-solving framework and develop multiple strategies.</p>	<p>Develop "Happy Numbers" (from Math Teacher, p. 618, November 1986).</p> <p>(R) Find and display all happy numbers less than 100.</p> <p>19 is a happy number. Let's see why.</p> $ \begin{array}{rcl} 19 & \rightarrow & 1 \times 1 + 9 \times 9 = 82 \\ 82 & \rightarrow & 8 \times 8 + 2 \times 2 = 68 \\ 68 & \rightarrow & 6 \times 6 + 8 \times 8 = 100 \\ 100 & \rightarrow & 1 \times 1 + 0 \times 0 + 0 \times 0 = 1 \end{array} $ <p>Since the sequence of numbers 19, 82, 68, 100, and 1 ends with the number 1, we say that the number 19 is a happy number.</p> <p>HAPPY NUMBERS</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td>1</td> <td>7</td> <td>10</td> <td>13</td> <td>19</td> </tr> <tr> <td>23</td> <td>28</td> <td>31</td> <td>32</td> <td>44</td> </tr> <tr> <td>49</td> <td>68</td> <td>70</td> <td>79</td> <td>82</td> </tr> <tr> <td>86</td> <td>91</td> <td>94</td> <td>97</td> <td></td> </tr> </table>	1	7	10	13	19	23	28	31	32	44	49	68	70	79	82	86	91	94	97	
1	7	10	13	19																		
23	28	31	32	44																		
49	68	70	79	82																		
86	91	94	97																			

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Applies and practises problem-solving skills in new situations. (cont'd)		<p>(E) Develop a program for "Happy Numbers", such as:</p> <pre> 10 PRINT "HAPPY NUMBERS" 20 PRINT 30 REM CHECK ALL NATURAL NUMBERS LESS THAN 100 40 FOR X = 1 TO 99 50 Z = X 60 REM GENERATE SIX TERMS 70 FOR N = 1 TO 6 80 IF Z > = 100 THEN 170 90 IF Z > = 10 THEN 200 100 Z = Z * Z 110 NEXT N 120 REM CHECK FOR HAPPY NUMBERS 130 IF Z = 1 THEN PRINT X, 140 NEXT X 150 GOTO 220 160 REM ISOLATE THE DIGITS OF THE 3-DIGIT NUMBERS 170 H = INT (Z/100):T = INT (Z/10) - H * 10:U = Z - (H * 100 + T * 10) 180 Z = H * H + T * T + U * U: GOTO 110 190 REM ISOLATE THE DIGITS OF THE 2-DIGIT NUMBERS 200 T = INT (Z/10):U = Z - T * 10 210 Z = T * T + U * U: GOTO 110 220 END </pre>	

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations.	<p>An equal emphasis should be placed on the various strategies for computing. Single-digit basic facts should be drilled on a regular basis through activities such as timed challenges or games. Paper-and-pencil strategies should be used to develop an understanding of sub-concepts such as re-grouping, borrowing or place value. Long and tiresome paper-and-pencil drill is discouraged.</p> <p>Estimation should be done on a daily basis. Recognition of appropriate situations for estimates, determining how precise an estimate should be for a given situation, and knowing when a computed answer is possible, are among skills to be emphasized.</p> <p>Mental computation involves using natural and easy strategies to compute exact answers. Strategies should be identified and shared as they evolve.</p> <p>Calculators should be used to develop understanding to investigate patterns, and to perform (tedious) computations that do not enhance understanding.</p>	<p>The following BASIC program should give your students an opportunity to practise their skills at estimation. The program can easily be modified to provide practice with any of the operations. It will work with both whole numbers and decimals.</p> <p>Each player takes turns inputting a value of their choice. Then each is asked to estimate the results of using a particular operation on the two numbers. The estimate closest to the actual answer is declared the winner. This program is written in a simple form of BASIC and should run on any computer that uses BASIC.</p> <p>(Arithmetic Teacher, Vol. 32, #5, January 1985.)</p>	

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations. (cont'd)			<pre> 5 REM ESTIMATION GAME 10 LET OP\$ = "SUM" 15 PRINT "IN TURN EACH PLAYER WILL ENTER" 20 PRINT "A NUMBER OF THEIR CHOICE" 25 PRINT 30 PRINT "THEN EACH PLAYER WILL ENTER AN" 35 PRINT "ESTIMATE OF THE ..OP\$;" OF THE NUMBERS." 40 PRINT 45 PRINT "THE CLOSEST ESTIMATE WINS!" 50 PRINT 55 INPUT "FIRST PLAYER'S NUMBER = >":A 60 INPUT "SECOND PLAYER'S NUMBER = >":B 65 PRINT 70 INPUT "FIRST PLAYER'S ESTIMATE...":A1 75 PRINT 80 INPUT "SECOND PLAYER'S ESTIMATE...":B1 85 LET C = A + B:REM FINDS SUM 90 IF C - A1 < C - B1 THEN 115 95 PRINT 100 PRINT "PLAYER #2 WINS!" 105 GOTO 120 110 PRINT 115 PRINT "PLAYER #1 WINS!" 120 END </pre>

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations. (cont'd).</p> <p>Line 10 can be changed to indicate a different operation. By changing line 10 to <code>LET OP\$ = "PRODUCT"</code>, the instructions in line 35 tell the user to estimate the answer to a multiplication problem. If line 10 is changed, then the operation in line 85 must also change. If <code>OP\$ = "PRODUCT"</code>, then line 85 must be <code>LET C = A*B.</code></p> <p>The following lines can be added to make the program repeat.</p> <pre> 120 PRINT 125 INPUT "AGAIN (Y/N)?": Y\$ 130 IF Y\$ < > "Y" THEN 200 135 GOTO 40 200 END </pre>			

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>A. Whole Numbers</p> <ol style="list-style-type: none"> Maintains previously developed skills with whole numbers (operations, order of operations, evaluation of expressions, prime numbers, factorization, divisibility). Finds the greatest common factor of a given set of numbers. 	<p>Review Grade 7 notes on whole numbers Objectives #7, 8 and 10 as methods to determine the GCF.</p> <p>Use calculators and computers to determine the GCF of a set of numbers.</p> <p>Use GCF's to reduce fractions to lowest terms.</p> <p>Methods of finding GCF's include:</p> <ol style="list-style-type: none"> listing the factors of the numbers, and finding the common factors. 	<p>The listings on the right can be used to find GCF's on a computer. The first listing will determine the GCF of two numbers. Students could type the program onto the computer and use it.</p> <p>(E) An enrichment activity could be to have students either write their own program or improve the first listing.</p> <p>(E) Another enrichment activity could be to change the first listing so that the computer will determine the GCF of more than two numbers. Listing (b) is a modification of listing (a); the changes and additions will allow the computer to find the GCF of four numbers.</p>	<p>e.g., Darren wants to cut a 32 cm by 20 cm birthday cake into square pieces. What is the largest size he can cut? How many square pieces does he cut?</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 2. Finds the greatest common factor of a given set of numbers. (cont'd)	ii) finding the prime factors of the numbers and then determining which factors are common. Example: $\begin{array}{r} 24 = 2 \times 2 \times 2 \times 3 \\ 32 = 2 \times 2 \times 2 \times 2 \times 2 \\ \text{GCF} = 2 \times 2 \times 2 = 8 \end{array}$ Simplify: $\frac{24}{32}$ $\frac{24 \div 8}{32 \div 8} = \frac{3}{4}$ (simplest terms)	BASIC Listing: 5 REM GCF FOR 2 NUMBERS 10 HOME 20 INPUT "FIRST NUMBER?";A 30 INPUT "SECOND NUMBER?";B 40 IF A>B THEN N=B:GOTO 60 50 N = A 60 FOR X=1 TO N 70 Y = A/X : Z = B/X 80 IF Y = INT(Y) AND Z = INT(Z) THEN CF = X 90 NEXT 100 PRINT:PRINT 110 PRINT "THE GREATEST COMMON FACTOR IS";CF 120 END	

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 2. Finds the greatest common factor of a given set of numbers. (cont'd)			<pre> 5 REM GCF FOR 4 NUMBERS 10 HOME 20 INPUT "FIRST NUMBER?";A 22 N = A 25 INPUT "SECOND NUMBER?";B 28 IF A > B THEN N = B 30 INPUT "THIRD NUMBER?";C 32 IF C < N THEN N = C 35 INPUT "FOURTH NUMBER?";D 40 IF D < N THEN N = D 50 FOR X = 1 TO N 70 Y1 = A/X; Y2 = B/X; Y3 = C/X; Y4 = D/X 80 IF Y1 = INT(Y1) AND Y2 = INT(Y2) AND Y3 = INT(Y3) AND Y4 = INT(Y4) THEN CF = X 90 NEXT 100 PRINT:PRINT 110 PRINT "THE GREATEST COMMON FACTOR IS";CF 120 END </pre> <p>(Holtmath 7 Teacher's Edition - GCF/LCM, p. 144-145.)</p>

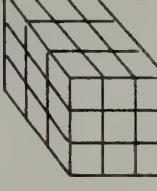
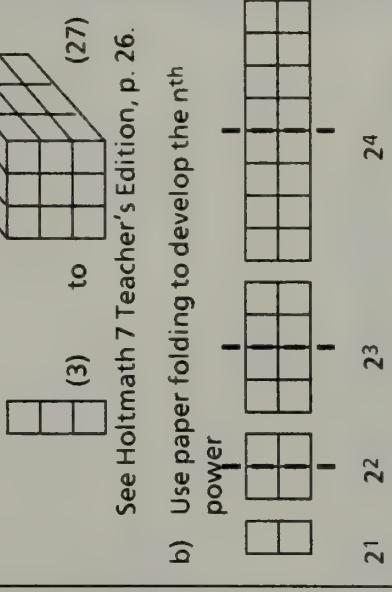
NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>A. Whole Numbers</p> <p>3. Finds the lowest common multiple of a given set of numbers.</p>	<p>Apply concepts from Grade 7 Whole Numbers Objectives #7, 8, and 10 as strategies to attain the LCM.</p> <p>Try a guess and check strategy to find LCM.</p> <p>Relate LCM to lowest common denominator to assist in making equivalent fractions for operational purposes. Stress the meanings rather than the acronym (GCF, LCM).</p>	<p>(E) Program a computer to <u>select</u> the LCM of given numbers. (See previous objective.)</p> <p>(E) The listing at right could be improved or could be changed to find the LCM of more than two numbers (as was done for GCF's).</p> <p>(E) Utilize the relationship between the GCF and LCM of two numbers (A and B): $\text{GCF} \times \text{LCM} = A \times B$.</p>	<p>BASIC Listings:</p> <pre> 5 REM LCM OF 2 NUMBERS 10 HOME 20 INPUT "FIRST NUMBER?";A 30 INPUT "SECOND NUMBER?";B 40 IF A < B THEN N = B:P = A:GOTO 60 50 N = A: P = B 60 X = 0 70 X = X + 1 80 Y = N * X/P 90 IF Y = INT(Y) THEN LCM = N*X: GOTO 110 100 GOTO 70 110 PRINT:PRINT 120 PRINT "THE LOWEST COMMON MULTIPLE IS"; LCM 130 END </pre> <p>Software: Holtmath 7 LCM, pp. 146-147.</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>A. Whole Numbers</p> <p>4. Understands and uses the terms exponent, base, power, squared and cubed and the n^{th} power of a number.</p>	<p>Write the value of a power with a whole number base and exponent (see Grade 7 notes on Whole Numbers Objective #4).</p> <p>a) Use blocks as manipulatives to develop concepts of making squares</p>  <p>(3) to (9)</p> <p>or making cubes</p>  <p>(3) to (27)</p> <p>See Holtmath 7 Teacher's Edition, p. 26.</p> <p>b) Use paper folding to develop the n^{th} power</p>  <p>OR</p> <p>If $2^2 = \square\square$ and $2^n = \square\square\square\square$ then discuss the value of n.</p>	<p>(E) Continue development to encompass square roots using blocks. Have students construct squares; use area and dimensions to develop an understanding of a square root.</p> <p>(E) Develop law of multiplication with exponents.</p>	<p>Using grid paper on the overhead ask students to use their calculators to guess and check and find n.</p> <p>e.g., if $2n = 1064$ $n = ?$</p> <p>Use calculators in a 'mad minute' activity to reinforce concept.</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>A. Whole Numbers</p> <p>5. Demonstrates the need for scientific notation.</p>	<p>(See Holtmath 8 Teacher's Edition, pp. 56-57. Journeys in Math 8 TRM, p. 71.)</p> <p>(R) Round off numbers to approximate answers. Multiply whole numbers and count zeros to avoid writing all of the zeros.</p>	<p>e.g., $87000 \times 5670 = n$</p> <p>a) Discuss calculator output such as:</p> <ul style="list-style-type: none"> • 49 329 000 E where multiplication result is too large for calculator to handle • 4.9329 8 represents scientific notation as 4.9329×10^8 <p>b) Use computer to type the following command – PRINT 670 924 000 – discuss its output of $6.70\ 924E + 08$.</p>	

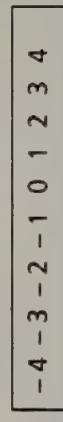
NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology		
<p>A. Whole Numbers</p> <p>6. Writes numbers in scientific notation, and scientific notation numbers in standard form (limit: positive exponents).</p>	<p>(See Holtmath 8 Teacher's Edition, pp. 56-57. Journeys in Math 8 TRM, p. 71.)</p>	<p>Students work in pairs to make and use flashcards to convert numbers from scientific notation to standard form and vice versa.</p> <p>e.g.,</p> <table border="1" data-bbox="487 587 629 968"> <tr> <td data-bbox="487 799 597 968">6×10^8 FRONT</td> <td data-bbox="597 587 629 799">600 000 000 BACK</td> </tr> </table>	6×10^8 FRONT	600 000 000 BACK	
6×10^8 FRONT	600 000 000 BACK				

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>B. Integers</p> <ol style="list-style-type: none"> Maintains previously developed skills with integers (need for integers, concept of integers, ordering of integers, demonstrates addition of integers with manipulatives). States the additive inverse of any integer. 	<p>See Grade 7 notes.</p>	<p>Use the terms "additive inverse" and "opposite" interchangeably. Discuss temperature, bank account and above/below sea level to develop the concept of additive inverse/opposite.</p> <p>Students should understand that the additive inverse is a tool for computation.</p> <p>(Number + Additive Inverse = 0)</p> <p>("Concrete Development" Journeys in Math 7 TRM, p. 302.)</p>	<p>Holtmath 7 Courseware, Disk 5, Additive Inverse.</p> <p>(R) Use a strip of paper to develop a number line.</p> <p>e.g., </p> <p>Fold on zero, and find the additive inverse.</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>B. Integers</p> <p>3. Uses concrete manipulatives to demonstrate the subtraction, multiplication and division of integers.</p>	<p>Concrete development (subtraction). (Journeys in Math 7 TRM, pp. 304-305.)</p> <p>Extend the idea of changes and coloured chips to multiplication and division.</p> <p><u>Multiplication and Division</u></p> <p>Emphasize the meaning of the negative sign (opposite). Give each student a red (-) and a black (+) chip. Ask the students to replace the black chip with the opposite colour (one negative sign). What would happen if they were asked to replace the chip with the opposite colour two times (two negative signs)? Three times? Four times? The notion that pairs of opposites (negative signs) cancel each other should immerge.</p>	<p>Begin by using the black chips (+) to demonstrate multiplication as repeated addition and division as repeated subtraction. The colour of the chips to be used in the demonstration will change as integral values are introduced and will be determined by the number of negatives (opposites) in the question (e.g., 2×-3 means one opposite, or that red chips will be used in the demonstration).</p> <p>Note: Students must write a number sentence to describe each operation or activity.</p>	<p>Holtmath 7 Teacher's Edition, pp. 274-275.</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>B. Integers</p> <p>4. Performs the operations of addition, subtraction, multiplication and division with integers using paper-and-pencil algorithms, estimation, mental computation and a calculator.</p>	<p>Concrete activities should be extended to determine strategies for computing without manipulatives. As students write number sentences describing concrete operations, they should be looking for patterns that would enable them to perform the operations without the manipulatives.</p>	<p>(E) Journeys in Math 7 Teaching Aids.</p> <p>Games: 21 (p. 81) 23 (p. 82) 24 (p. 82)</p> <p>An equal emphasis should be placed on mental computation, pencil-and-paper computation, estimation and using the calculator. Timed activities and games may be used to encourage mental facility.</p>	<p>Students should be taught how to compute with integers on a calculator. Because calculators may vary in their operating systems, some time should be taken to investigate the change sign key and to verify the operating system of the calculator. Students should be encouraged to verbalize how their own calculators work when operating with integers.</p> <p>Additional: See Journeys in Math 7 TRM, p. 309.</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>C. Rational Numbers</p> <ol style="list-style-type: none"> Maintains previously developed skills with decimal numbers (place value, operations, ordering, rounding, order of operations). Maintains previously developed skills with fractional numbers (concept of a fraction, equivalent fraction, basic fraction, mixed numbers, improper fraction, ordering fractions, concrete operations with fractions, order of operations). Writes the multiplicative inverse (reciprocal) of a fraction, whole number or integer. 	<p>Develop <u>reciprocals</u> for <u>proper fractions</u> only. Students should understand how the reciprocal/multiplicative inverse acts as a tool (number \times reciprocal = 1).</p> <p>Get students to realize that dividing by 2 is the same as taking one half of an item. Progress to show $\div 3 = \times 1/3$, $\div 7 = \times 1/7$. Discuss the relationships between 2 with 1/2, 3 with 1/3, and 7 with 1/7. Extend to what happens to 2/3.</p>		

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>C. Rational Numbers</p> <p>4. Performs the operations of addition, subtraction, multiplication and division with fractions (limit: positive rationals).</p>	<p>Review the four operations with fractions at a concrete level. Students should be required to write number sentences to describe the concrete operations while looking for patterns that would enable them to perform the operations without using manipulatives. The fractions used to perform initial paper-and-pencil or mental operations should be relatively simple and should easily be related to a concrete manipulative (especially if difficulties occur). Estimation of answers to computations with fractions should be encouraged continuously.</p>	<p>(R) Students should continue to use manipulatives or pictorial presentation until they are able to perform operations formally.</p> <p>(R) Encourage the students to make and use fraction slide rules.</p> <p>(E) Develop games like fraction dominos.</p> <p>(E) Extend strategies developed to include mixed numbers.</p>	<p>Holtmath 7, Disk 3.</p>

5. Demonstrates the need for rational numbers (e.g., $-3 \div 2 = ?$ No answer is possible without a set of rational numbers).

Discuss examples of uses of rational numbers:

e.g., **Measurement**
Banking – deposits, withdrawals,
overdrafts
Business – profits, losses

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>C. Rational Numbers</p> <p>6. Recognizes rational numbers as all numbers that can be written in the form:</p> $\frac{a}{b}$ where $b \neq 0$.	<p>It is important that students understand that numbers have many different but equivalent forms. Have the students construct a fraction tape (a strip of paper folded in halves, quarters, eighths, thirds, sixths, etc.) and mark the folds as one would mark a number line (zero in the middle, with rational numbers describing the folds on each side). Then ask the students to write the decimal and, if possible, integral equivalents to the rational numbers.</p> <p>Discuss the fact that any number (regardless of its form) that <u>can be</u> written as a fraction or a/b where $b \neq 0$ form, is by definition a rational number. Use a calculator to explore the various patterns formed by fractions and their equivalent decimal forms (e.g., $1/7$, $2/7$, etc., $1/9$, $2/9$, etc., $1/11$, $2/11$, etc.).</p>	<p>(E) Are there numbers that <u>do not</u> have fractional equivalents? What are these numbers (irrational numbers)? How are they generated? (See <i>Journeys in Math 8 TRM</i>, p. 157.)</p> <p>(R) Use calculator to make an equivalent form to explore differences.</p> <p>(E) See <i>Journeys in Math 7 TRM</i>, pp. 174 – 175.</p>	
<p>7. Compares and orders rational numbers using $<$, $>$ or $=$.</p>	<p>Use the fraction tape (Objective #6) to compare and order rational numbers. It should be made clear to students that on a number line, numbers on the right are larger than numbers on the left (e.g., $1 > -3$).</p>		

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
C. Rational Numbers 8. Uses a number line to demonstrate the relationship between whole numbers, integers, fractions and rationals.	Extend the fraction tape. Discuss the relationship between whole numbers, integers, fractions and rationals.		

RATIO AND PROPORTION

Grade 8

Objective	Clarification or Example	Elective Suggestions
<p>1. Applies and practises problem-solving skills in new situations.</p>	<p>The intent is to develop non-routine problem-solving skills in every student. Each student should understand the concepts of the problem-solving framework and develop multiple strategies.</p> <p>(Refer to Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985.)</p>	<p>MAC Courseware 9A Disk B. Ratio Rendezvous 9B.</p> <p>(This should be used for enrichment.)</p> <p>(E) It is important to clarify that fractions are ratios but ratios are not fractions!</p> <p>e.g., $400/700 \rightleftharpoons 4/7$ ratio $400/700 = 4/7$ fraction</p> <p>Note:</p> <p>1 girl/2 boys Ratio 1 as to 2 Number of girls not $1/2$</p> <p>(R) For concrete development of these skills see detailed explanation at the Grade 7 level.</p>
<p>2. Maintains previously developed skills (understands and constructs ratios, equivalent ratios; finds missing element of a proportion, percent as a ratio, percents as decimals, percents of numbers; and expresses one number as a percent of another).</p>	<p>Encourage the development of verbal activities (mental facility) such as "mad minute" to reinforce maintenance skills.</p> <p>e.g.,</p> <p>(i) "Give two equivalent ratios for..." (ii) "Express this ratio in three ways..." (iii) Find the unknown $3/4 = x/20$ using two methods. (iv) Advertisements which illustrate percentage discounts. (v) Estimate percentages of test scores and check using a calculator.</p>	<p>Students use software from Mac Courseware 8 – Disk B program, 9B – Ratio Rendezvous. This will help students to develop their own method of identifying equivalent ratios.</p>

RATIO AND PROPORTION

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>2. Maintains previously developed skills (understands and constructs ratios, finds equivalent ratios; finds missing element of a proportion, percent as a ratio, percents as decimals, percents of numbers; and expresses one number as a percent of another). (cont'd)</p>	<p>A practical example is mixing photography chemicals, e.g.,</p> <p>a) 1 part of Water : 9 parts of Developer b) 2 parts of W : 18 parts of D c) 3 parts of W : ? parts of D.</p> <p>Students can investigate other examples.</p> $\frac{1}{9} = \frac{2}{18}$ <p>Students should be encouraged to suggest other practical examples.</p>	<p>(E) Students can investigate trends in the stock market.</p> <p>If a written report is required, the criteria for evaluation must be clearly stated.</p>	
			<p>Discuss what a percent greater than 100 means and its practical occurrences (stock markets, growth in industry, retail sales).</p> <p>a) Use 10×10 grids to demonstrate percents greater than 100. Have students shade the grids to show, for example, 120% (one complete 10×10 grid and 20 squares from a second grid). Note that 120% is a number larger than 1 but smaller than 2.</p>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology																				
3. Gives examples of ratios involving situations where the equivalent percent is greater than 100. (cont'd)	<p>b) Use a chart similar to the one below.</p> <table border="1" data-bbox="409 1080 781 1644"> <thead> <tr> <th data-bbox="409 1080 487 1644">Cost Price</th><th data-bbox="487 1080 566 1644">Selling Price</th><th data-bbox="566 1080 644 1644">Profit</th><th data-bbox="644 1080 781 1644">% Profit (x)</th></tr> </thead> <tbody> <tr> <td data-bbox="409 1080 487 1644">\$20</td><td data-bbox="487 1080 566 1644">\$25</td><td data-bbox="566 1080 644 1644">\$5</td><td data-bbox="644 1080 781 1644">$\frac{5}{20} = \frac{x}{100} \therefore x = 25\%$</td></tr> <tr> <td data-bbox="409 1080 487 1644">\$20</td><td data-bbox="487 1080 566 1644">\$30</td><td data-bbox="566 1080 644 1644">\$10</td><td data-bbox="644 1080 781 1644">$\frac{10}{20} = \frac{x}{100} \therefore x = 50\%$</td></tr> <tr> <td data-bbox="409 1080 487 1644">\$20</td><td data-bbox="487 1080 566 1644">\$40</td><td data-bbox="566 1080 644 1644">\$20</td><td data-bbox="644 1080 781 1644">$\frac{20}{20} = \frac{x}{100} \therefore x = 100\%$</td></tr> <tr> <td data-bbox="409 1080 487 1644">\$20</td><td data-bbox="487 1080 566 1644">\$60</td><td data-bbox="566 1080 644 1644">\$60</td><td data-bbox="644 1080 781 1644">$\frac{40}{20} = \frac{x}{100} \therefore x = 200\%$</td></tr> </tbody> </table> <p>Let students suggest other examples.</p>	Cost Price	Selling Price	Profit	% Profit (x)	\$20	\$25	\$5	$\frac{5}{20} = \frac{x}{100} \therefore x = 25\%$	\$20	\$30	\$10	$\frac{10}{20} = \frac{x}{100} \therefore x = 50\%$	\$20	\$40	\$20	$\frac{20}{20} = \frac{x}{100} \therefore x = 100\%$	\$20	\$60	\$60	$\frac{40}{20} = \frac{x}{100} \therefore x = 200\%$	<p>4. Converts mixed numbers to percents and vice versa.</p> <p>Percents larger than 100 have equivalent mixed number forms. Extend the development from Objective 3 to include many examples of mixed numbers (more than one 10×10 grid) and their equivalent percents.</p>	<p>Understanding the operation is a necessary requirement before using a calculator or computer. Before permitting (or instructing) students on using the percent key on the calculator, ask the students to describe (in a paragraph) how the percent key on their own calculator works. In other words, describe the algorithm that the engineer (builder) used to "program" the calculator.</p>
Cost Price	Selling Price	Profit	% Profit (x)																				
\$20	\$25	\$5	$\frac{5}{20} = \frac{x}{100} \therefore x = 25\%$																				
\$20	\$30	\$10	$\frac{10}{20} = \frac{x}{100} \therefore x = 50\%$																				
\$20	\$40	\$20	$\frac{20}{20} = \frac{x}{100} \therefore x = 100\%$																				
\$20	\$60	\$60	$\frac{40}{20} = \frac{x}{100} \therefore x = 200\%$																				

RATIO AND PROPORTION

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>5. Given the percent determines the missing value in applications such as discounts, increases, decreases, or sales tax.</p>	<p>Groups of students research discounts, increases, decreases or sales tax using newspaper advertisements. Classroom discussion can follow, stressing mental computation and estimation. Use the calculator to check, e.g., (best deal - discount).</p> <p>Ask students to collect daily newspaper articles. Discuss the meanings of: 50% more, 30% discount, 5% sales tax. Follow up with students creating their own advertisements.</p>	<p><u>Item:</u> Suit <u>Reg. Price:</u> \$150 <u>% Discount:</u> 30% i.e., $30 \div 100$ <input data-bbox="640 95 676 150" type="text" value="%"/> <u>Amount of Reduction:</u> $0.3 \times 150 = 45$ <u>Sale Price:</u> $150 - 45 = 105$</p> <p><u>Item:</u> Stereo <u>Reg. Price:</u> \$280 <u>% Discount:</u> 15% i.e., $15 \div 100$ <input data-bbox="958 95 994 150" type="text" value="%"/> <u>Amount of Reduction:</u> $0.15 \times 280 = 42$ <u>Sale Price:</u> $280 - 42 = 238$</p>	<p>Note: <input data-bbox="1178 418 1214 473" type="text" value="%"/> Represents the percent key on calculators.</p>

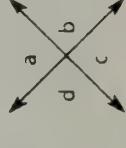
RATIO AND PROPORTION

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology										
<p>6. Understands and writes rates as the comparison of two numbers with different units (e.g., 15 km/2h or 3 items/\$1).</p>	<p>(Holtmath 8 Teacher's Edition, pp. 140-141. Journeys in Math 8 TRM, p. 178.)</p> <p><u>Batting Statistics</u></p>	<p>(E) Use statistics in sports to calculate the m.v.p. (most valuable player) for a particular team or sport.</p> <table border="1" data-bbox="572 676 745 1057"> <thead> <tr> <th data-bbox="572 676 635 877">At Bat</th> <th data-bbox="635 676 745 877">Hits</th> </tr> </thead> <tbody> <tr> <td data-bbox="651 983 698 1057">Tom</td> <td data-bbox="698 983 745 1057">60</td> </tr> <tr> <td data-bbox="651 1004 698 1057">Dick</td> <td data-bbox="698 1004 745 1057">40</td> </tr> <tr> <td data-bbox="651 1025 698 1057">Harry</td> <td data-bbox="698 1025 745 1057">50</td> </tr> <tr> <td data-bbox="698 983 745 1057"> </td> <td data-bbox="698 1004 745 1057">20</td> </tr> </tbody> </table> <p>(See Journeys in Math 7 TRM, p. 219.)</p>	At Bat	Hits	Tom	60	Dick	40	Harry	50		20	<p>(Holtmath 8 Teacher's Edition, pp. 136-139. "Alternative Teaching Strategies". Journeys in Math 8 TRM, p. 170.)</p> <p>(E) Students compare sizes of a selected product to determine which is the most economical to buy. The intent is for students to find the unit rates (comparison shopping).</p> <p>(Holtmath 8 Teacher's Edition, pp. 141-145.)</p>
At Bat	Hits												
Tom	60												
Dick	40												
Harry	50												
	20												
<p>7. Writes proportions involving rates.</p> <p>8. Finds the missing element in a proportion involving rates.</p>													

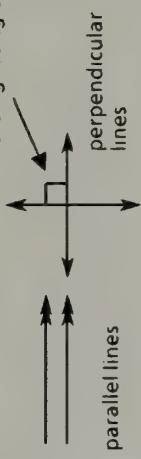
MEASUREMENT AND GEOMETRY

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>1. Applies and practises problem-solving skills in new situations.</p> <p>2. Maintains previously developed skills (linear, area, volume, capacity and mass units of measure; uses geometric tools to measure line segments and angles and to construct geometric designs; transformational geometry).</p> <p>3. Understands and uses the terms perpendicular and parallel lines.</p>	<p>(Holtmath 8 Teacher's Edition, pp. 178-181. Journeys in Math 8 TRM, p. 234.)</p>	<p>(E) Explore the relationships between angles of intersecting lines.</p> <p>e.g.,</p> 	<p>$a = c$, $d = b$, $b + c = 180^\circ$</p>

MEASUREMENT AND GEOMETRY

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3. Understands and uses the terms perpendicular and parallel lines. (cont'd)	The intent of this objective is to investigate the attributes (characteristics) of polygons which will lead to a definition of a polygon (a plane closed figure whose sides are line segments). Further "investigation" of the attributes (e.g., measures of sides, congruence of sides and/or angles, measure of interior angles, number or length of diagonals) can lead to the classification of the polygons (according to number of sides or as regular) which is objective #5.	<p>Give pictorial diagrams to show difference between perpendicular and parallel lines.</p> 	<p>The following LOGO procedure is an example of a program that may be generated by students. This procedure will allow the construction of various polygons by "inputting" the length of a side, and the measure of the interior angle.</p> <p>For example, SHAPE: 40 60 will result in a square being plotted on the screen.</p> <p>Students should be encouraged to change the measures of the angles (and the sides, in cases where the shape is too small or too large for the screen) to construct varying shapes or polygons. It must be noted that this procedure will not stop unless it is interrupted (with a CONTROL-G command in the case of Apple LOGO). The results should be tabulated: length of side; size of</p>
4. Draws or sketches various polygons using tools such as a computer, compass, straightedge, ruler, protractor.	<p>Students should be asked to place a number of dots (points) randomly on a paper (plane). Keep the number of points small (2-10). Then ask students to connect the points to form various figures. Discuss the outcome by grouping (classifying) the figures using the following points as a guide:</p>	<p>The following points may be used as a guide in the investigations and subsequent discussions:</p> <ol style="list-style-type: none"> congruent sides congruent interior angles both congruent sides and angles are conditions for a regular polygon number of diagonals relationship of the perimeter of a regular polygon to the length of its diagonal. 	

MEASUREMENT AND GEOMETRY

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
4. Draws or sketches various polygons using tools such as a computer, compass, straightedge, ruler, protractor. (cont'd)	<ul style="list-style-type: none"> a) the figure must be closed (have an interior and an exterior) b) line segments (not curved lines) define the sides of a polygon c) the minimum number of points required to close a figure is 3. <p>Further investigation will lead to the classification of the polygons according to the number of sides.</p>	<p>Many techniques or instruments may be used to perform the actual sketches or constructions. Traditional tools such as the compass, protractor or ruler have "built-in" mathematics concepts but their use may be tedious. The computer will allow students (after learning to use a graphics program or a program such as LOGO) to generate many and varying shapes in a short period of time or to "instruct" a computer to construct a polygon. (For more information on using a LOGO program see <i>Journeys in Math 8</i> TRM, pp. 91, 94-95.)</p>	<p>angle (note that the angle is not the interior angle, but the supplement of the interior angle ($180 - \text{ANGLE}$) in this procedure); and the shape of the figure (polygon or start).</p> <p><u>LOGO Listing</u></p> <pre>TO SHAPE: SIDE ANGLE FD: SIDE RT: ANGLE SHAPE: SIDE: ANGLE END</pre>

MEASUREMENT AND GEOMETRY

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>5. Identifies and classifies polygons according to the number of sides (limit: decagon).</p> <p>6. Investigates triangles by examining attributes such as measure of angles, measure of sides and lines of symmetry.</p>	<p>See Objective #4.</p> <p>Investigate triangles by having students construct a number of random triangles. Discuss their similarities and differences and the different attributes (length of sides, measure of angles, number of lines of symmetry) that may be used to classify the triangles (<u>before</u> assigning names to the classifications).</p>	<p>(E) Students could investigate to find the names of some polygons with more than 10 sides.</p> <p>(E) See Holtmath 8 Teacher's Edition, pp. 186-187, "Rigid and Non-Rigid Shapes".</p> <p>(See Holtmath 8 Teacher's Edition, pp. 184-185. Journeys in Math 8 TRM, pp. 242-243.)</p>	<p>Problem Solving: Explore tangram-type puzzles.</p> <p>Computer: Holtmath 7, Software Disk 4, "Angles of Regular Polygons".</p>
<p>7. Investigates quadrilaterals by examining attributes such as measure of sides, measure of angles, lines of symmetry and diagonals.</p>		<p>A discovery approach is encouraged. Either provide to students, or have them construct (from black line masters) a "quadrilateral kit". This kit should contain a number of varying shapes that are large enough to measure and manipulate: irregular quadrilaterals, trapezoids, parallelograms, rhombi, rectangles and squares. Discuss the various attributes of the shapes and then have the students group the shapes according to those attributes. Note that some of the quadrilaterals can fit into several categories.</p>	<p>(E) Journeys in Math 7 TRM, p. 145.</p>

MEASUREMENT AND GEOMETRY

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
7. Investigates quadrilaterals by examining attributes such as measure of sides, measure of angles, lines of symmetry and diagonals. (cont'd)	<p>e.g., Quadrilaterals that have: four sides; (at least) one pair of parallel sides; opposite parallel sides; opposite sides are congruent (Can opposite sides of quadrilaterals be parallel without being congruent?); congruent opposite angles; opposite parallel sides and at least one right angle; number of lines of symmetry; number of diagonals; congruent diagonals; diagonals bisect each other; diagonals meet at right angles.</p> <p>Develop a classification scheme based on the properties of the quadrilaterals, which would relate quadrilaterals, trapezoids, parallelograms, rhombi and squares.</p> <p>Alternative developments:</p> <p>(Journeys in Math 7 TRM, pp. 144-145. Journeys in Math 8 TRM, pp. 244-245. Holtmath 8 Teacher's Edition, pp. 194-195.)</p>		

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
8. Adds, subtracts, multiplies and divides using SI units of measure.	<p>Only commonly used units (km, m, cm, and mm) should be used to develop this objective. The operations with units of measure should be developed in the context of real problems (e.g., perimeter and area). To add, subtract, multiply or divide measures, the units must be common (the same) or else the result will not have a standard meaning (e.g., $3 \text{ m} + 4 \text{ cm} = ?$ or $2 \text{ cm} \times 5 \text{ mm} = ?$).</p>	<p>(E) Explore the relationship between, for example, 1 m^2 and cm^2.</p> <p>(E) Explore the unit of area hectare (ha).</p>	<p>Multiplication (or its inverse-division) of measures is a difficult concept. Students should be taught that whenever they see, for example, $\text{cm} \times \text{cm}$ this does not mean cm "times" cm but, rather, is an expression of area (a count of square centimetres (cm^2)).</p> <p>This objective may be developed within the context of objectives #9 (perimeter) and #10 (area).</p>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
9. Understands and uses formulas as indirect measures of the perimeter of polygons (includes regular polygons).	<p>Formulas are indirect strategies for calculating measures that can sometimes be directly measured. Students should first have experiences with the direct measures before attempting to devise a strategy (or formula) for computing the measure. For perimeter this experience may be using a string (and a ruler) to determine the distance around different polygons; then using only a ruler to measure the sides and then determining the sum of the sides. Students should note that the order in which the measures are taken or summed, does not affect the result. Students should verbalize a strategy before attempting to generalize it in the form of a formula. For example, "the perimeter of a rectangle is two lengths added to two widths because . . ." The generalization or the formula should be the last stage of the development and may be used to expedite the process of determining a perimeter. The use of formulas may not be appropriate for all students.</p>	<p>(R/E) Journeys in Math 8 TRM, pp. 80-81.</p> <p>Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985. p. 47, #6.</p>	

MEASUREMENT AND GEOMETRY

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
10. Understands and uses formulas as indirect measures of the area of polygons (triangles, all parallelograms and trapezoids).	<p>Formulas are indirect strategies for calculating measures that can sometimes be directly measured. Students should first have experiences with the direct measures before attempting to devise a strategy (or formula) for computing the measure. For area, this experience is counting the number of squares that can be found within a closed region. Transparency grids ($\text{cm} \times \text{cm}$) may be used to determine the areas of both regular- and irregular- shaped objects. The measures should then move on to rectangles where it should be discovered that, the number of squares in one row corresponds to the length, and the number of rows corresponds to the width. Hence, the product of the length and the width determines area. Students should be encouraged to verbalize the strategy before generalizing it. Areas of other polygons (e.g., triangle, parallelogram) are variations of the rectangle.</p> <p>(See Holtmath 8 Teacher's Edition, pp. 74-82. Journeys in Math 8 TRM, pp. 83-87.)</p>		<p>Computer: Mathematics Activities Courseware 8, "Metric Mysteries", Disk C, Houghton Mifflin.</p>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>11. Performs an experiment to determine the value of π and understands π as a ratio of the circumference of a circle divided by its diameter.</p> $(\text{i.e., } \pi = \frac{C}{d})$	<p>An introductory activity for this objective could be to draw different sizes of a particular polygon and then determine the ratio of the perimeter to the length of a diagonal (have the students make the measurements).</p> <p>e.g., For a set of squares, $\frac{\text{perimeter}}{\text{diagonal}} = 2.9$</p>	<p>(E) Use the library or other resources to gather information about the historical development of π.</p> <p>(E) Develop a strategy for finding the diameter of a circle when the centre is not given.</p> <p>(E) Journeys in Math 8 TRM, p. 82.</p>	<p>Problem Solving: By comparing measurements, the question of accuracy should arise. When is it important? When is an estimation appropriate?</p> <p>From this activity the students should find the principle that, if figures have the same shape, the ratios of the corresponding parts remain constant even though the size is different.</p> <p>Using the above activity the students might predict that $\frac{C}{d}$ will be a constant (because all circles have the same shape). By measuring various round objects (records, coins, jars, cans) using a string and a ruler (a metre stick, if necessary) and also keeping a record of the diameters and circumferences, the concept that π is a ratio of C to d should be discovered.</p>

MEASUREMENT AND GEOMETRY

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>12. Understands and uses the formula $C = \pi d$ as an indirect measure of the circumference of a circle.</p>	<p>Using the measurements from Objective #11, compare calculated circumference ($C = \pi d$) to measured circumference. The development of the formula as a strategy should flow directly from the previous topic.</p> <p>Given the radius, explore how to find the circumference. Two possibilities can be used $C = \pi d$ (double the radius and then utilize the formula), or $C = 2\pi r$.</p>	<p>Use a compass and grid paper and have students draw a circle with a specified radius. By counting squares, estimate the area. Compare estimates.</p>	<p>Computer: Holtmath 7 Software, Disk 1, "Circumference of Circles".</p>
<p>13. Uses the formula $A = \pi r^2$ to indirectly determine the area of a circle given its radius or diameter.</p>	<p>Students show development of the formula $A = \pi r^2$.</p> <p>(E) Show that the area of a regular polygon ($A = \frac{1}{2} a n s$ where a is the apothem, n is number of sides, and s is the measure of one side) approaches the area of a circle ($A = \pi r^2$) as the number of sides increases.</p>		

MEASUREMENT AND GEOMETRY

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
14. Draws or sketches a right rectangular prism.	<p>From everyday objects such as a cereal box, or a kleenex box, discuss the relationship between sides. From this, students should be able to determine the characteristics of all right rectangular prisms. Prisms are easily drawn by using isometric paper or grid paper.</p> <p>(Journeys in Math 8 TRM, p. 98.)</p>	(E) Journeys in Math 8 TRM, p. 98.	
	<p>15. Understands and uses a formula as an indirect strategy for determining the volume of a right rectangular prism or a cube.</p> <p>A formula is an indirect strategy for calculating a measure that can sometimes be directly obtained. Students should have experiences with obtaining the measure directly before determining a strategy for computing the measure. Use centimetre cubes (sugar cubes will do if other manipulatives are unavailable) to determine the volume of rectangular prisms. Note that the number of cubes in one layer corresponds to the area of the base (length times width) and that the number of layers corresponds to the height.</p> <p>(See Holtmath 8 Teacher's Edition, pp. 91-93. Journeys in Math 8 TRM, p. 102.)</p>		

DATA MANAGEMENT

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>1. Applies and practises problem-solving skills in new situations.</p> <p>2. Maintains previously developed skills (understands the purpose of statistics; interprets data from tables and graphs; draws graphs).</p>	<p>See Grade 7 Data Management Objective #1.</p>	<p>a) Ask how numbers affect our lives. Use sources such as measurement, experimentation, observation, encyclopedias, newspapers, information services.</p> <p>b) Recap frequency tables, pictographs, bar graphs, line graphs, and circle graphs as ways of organizing and comparing data.</p> <p>Extended Content</p> <p>Utilize graphs from newspapers to display a bulletin board.</p>	<p>See Grade 7 Data Management Objective #1.</p> <p>Computer programs can display graphs. Houghton Mifflin, MAC 7, "Pie Graphs".</p> <p>(R) Explain strengths and weaknesses of each graph.</p> <p>(E) Use histograms, stem-leaf tables, box and whisker plots.</p>

DATA MANAGEMENT

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>3. Understands and uses the terms bias, sample and population.</p>	<p>Demonstrate sampling by taking a survey of students (e.g., from one row in a class) who are wearing running shoes, and then compare the sample to the population of the class.</p> <p>Discuss the bias by asking if this sample is likely to be representative of the school population (including the teachers) and of the town or municipality.</p> <p>Discuss how to obtain a useful sample in each case.</p>	<p>(R) Direct 3 or 4 pairs of students to complete a poll based on 10 responses on whether you like orange juice or apple juice. Ask students why results may vary.</p> <p>(E) Get students to conduct polls, interpret the results, and discuss problems that could arise.</p>	<p>Problem Solving: How are the fish in a lake counted?</p>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>4. Distinguishes between a survey and a census, understands when each is used and potential biases that may occur (survey).</p>	<p>A <u>survey</u> is a random poll of an identified group for the purposes of collecting data or acquiring information about some aspect of the group or area. A survey may be biased if the sample was not random (i.e., if the sample was not representative of the population) or if the sample included a population outside the identified group. For example, a survey to determine the most popular television program among junior high students would be biased if only one class of Grade 7 students were polled (only one age group was represented in the sample) or if some elementary or high school students were included in the sample.</p>	<p>(R) Select three students to be a class representative. Choose one row to see who they would elect. Choose another row. Predict how the whole class would vote. Compare results to voting of entire class.</p> <p>(E) Determine methods of taking a survey typical: (young vs old; male vs female; regional locations; high income vs low income).</p> <p>A census is a count or poll of an entire population and provides accurate information. A census is not always practical because of the size of the identified group and the time and costs associated with the gathering of information.</p> <p>Discuss the merits of a survey and a census and the situations or conditions that would be required to make a census or survey appropriate. When is accuracy more important (population of a school to determine per pupil grants) and when is knowing the information quickly more</p>	

DATA MANAGEMENT

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>4. Distinguishes between a survey and a census, understands when each is used and potential biases that may occur (survey). (cont'd)</p> <p>Extended Content</p> <ol style="list-style-type: none"> How does one conduct a survey: personal interviews? questionnaires? telephone calls? interviews? Discuss whether answers to surveys should be restricted or unlimited. Discuss problems associated with census. 	<p>important (deciding how many hot dogs and how many hamburgers should be purchased for a school function)?</p> <p>A discussion on how to ask questions could lead to realization that questions can influence the outcome of a survey?</p> <p>i.e., Do you feel the current tax hike is too high? What is your feeling on the proposed tax hike?</p>		<p>A data base like the Appleworks program can be used to sort information and/or 'pull out' the required information.</p>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
5. Recognizes the use and misuse of statistics in society (news reporting, census, polls, etc.).	<p>Statistics are used to predict outcomes but one questions their reliability as one finds incomplete, misleading, and misrepresented data. Find examples of how one uses statistics to one's advantage.</p> <p>e.g., Political survey polls which predict party support are often based on a limited sample which could create varying outcomes.</p>	<p>(R) Predicting the standing of a sports team (baseball, hockey, local team) based on its record in the last ten (x) games may be misleading. A team with a good record in the last ten (x) games may still end up last in the standings over the whole season.</p> <p>(E) Watch a newscast on TV to determine where statistics are used, how, why, and discuss their accuracy and timelines, etc.</p>	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>1. Applies and practises problem-solving skills in new situations.</p> <p>2. Maintains previously developed skills (variable, evaluation of expressions, concept of equality, plots on a coordinate plane).</p> <p>3. Identifies and combines like terms. (See Journeys in Math 8, pp. 346-347.)</p> <p>4. Uses formal procedures to solve equations of the form $x + a = b$, $ax = b$, $ax + b = c$, and $\frac{x}{a} = \frac{b}{c}$ (limit: positive rational numbers and integers).</p>	<p>(E) Use formal approach to explain answers to $x + 3x, 7n - 3n, 8y - 9y$. Expand into fractional variables.</p> <p>(E) Houghton Mifflin Mathematics Activities Courseware 8, Eckses and Ohzs, pp. 37-40.</p> <p>Review the concept of equality. (See Objective #6, Algebra Grade 7.) Formal algebraic techniques may be introduced by using the concept of balance and so a balance scale lends itself here. For example, a nickel and 2 one-gram masses balance with 7 one-gram masses. The mass of the nickel (represented by x) can be found by removing 2 one-gram masses from each side (the opposite operation).</p>	<p>Problem-solving strategies: experiment through the use of manipulatives; work backwards; break the problem into smaller parts; make and solve similar problems.</p>	

ALGEBRA

Grade 8

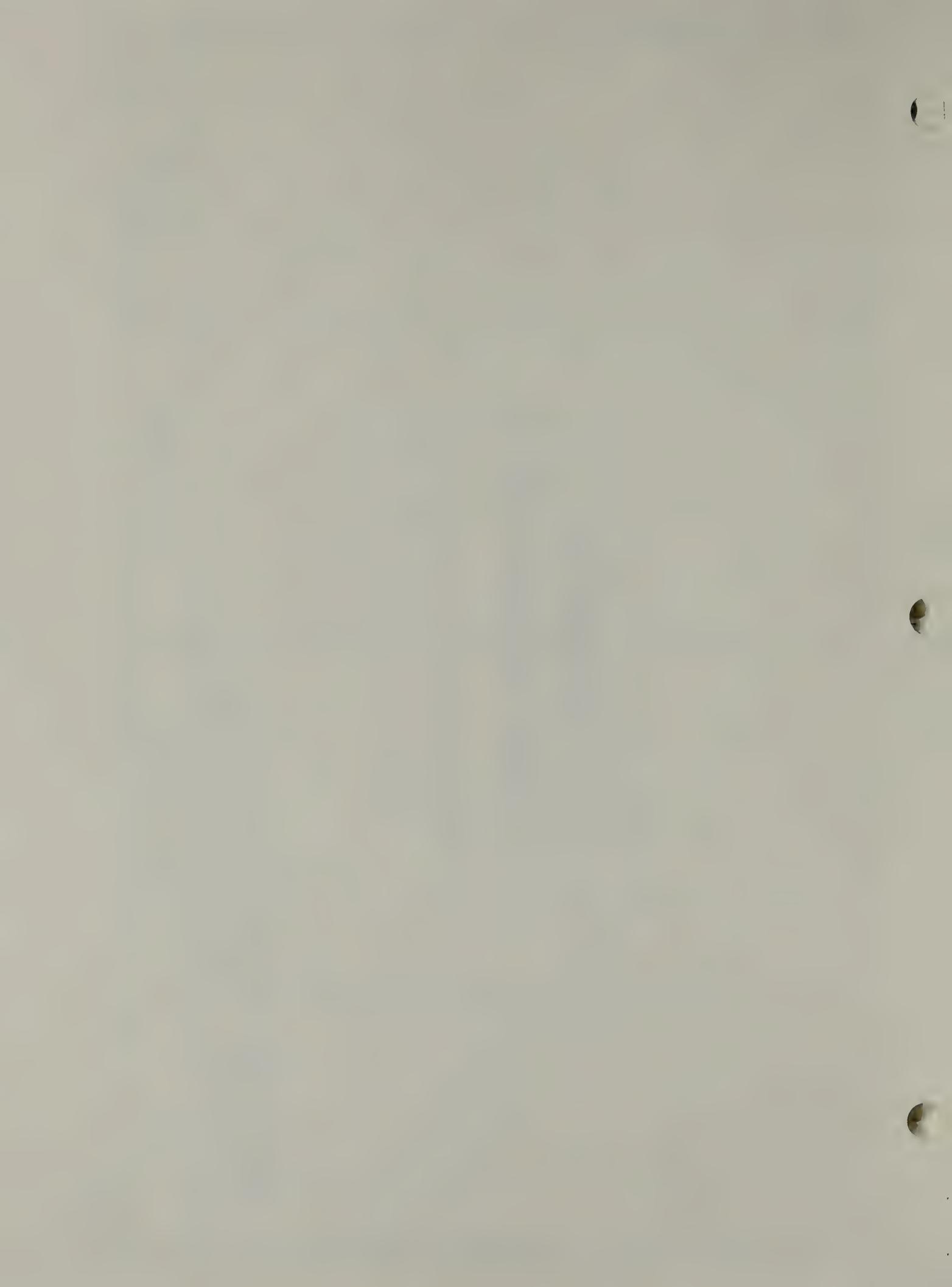
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
5. Verifies solutions to the equations.	<p>Review the concept of evaluating expressions.</p> <p>Verifying a solution to an equation is simply a matter of using the solution independently to evaluate the expression on each side of the equation to determine if in fact the LHS (left-hand side) is equal to the RHS (right-hand side).</p>	<p>(E) Estimate solutions by guess and check strategy.</p> <p>(E) Have students write their own nonsensical formulas and a funny story about what the variables represent. Exchange the formulae and stories within the class.</p> <p>For example: $E = 3 + 7a$ where E represents the number of elephants and a is the number of people with blue eyes.</p> <p>(E) Discuss the effect on a selected element of a formula if another element is doubled, tripled, etc.</p> <p>(R) Use real life situations where 3 apples and 4 oranges cost a certain amount of money. Give variations for 2 of the elements so the third element needs to be solved.</p>	
		<p>The intent of this objective is to de-mystify formulas. The few formulas presented in mathematics classes are usually taught as a means to an end (e.g., $d = rt$ is used to find distance, rate or time). The result is that little transfer occurs in knowing how to use other formulas in mathematics and other disciplines.</p> <p>A much more holistic view of formulas must be taken. The outcome of this objective should be that students will understand relationships among the variables, recognize the similarities (or differences) among those relationships (regardless of what kind of variables are used), and will know how to substitute and solve for missing elements.</p>	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
7. Generates a set of ordered pairs in a linear relation.	(See Holtmath 8 Teacher's Edition, pp. 290-295. Journeys in Math 8 TRM, pp. 308-309.)	<p>(E) Given the ordered pairs, determine the relation.</p> <p>(R) Give values for x as 0, 1, and 2 or numbers that are conducive to easy solution.</p> <p>(E) Develop a computer program to generate ordered pairs.</p>	<p>(R) Use a computer to give the pairs.</p> <p>5 REM ordered pairs 10 HOME 20 PRINT "This program will generate ordered pairs for relation rules in the form ax + b." 30 PRINT "You will be asked to give values of a and b, and then the computer will print values for ax + b by using a range of -10 to +10 for x." 40 INPUT "What value of a?";A 50 INPUT "What value of b?";B 60 HOME 70 PRINT "X", A; "X + ";B 80 PRINT "-----" 90 FOR X = -10 TO 10 100 PRINT X, A*X + B 110 NEXT 120 END</p>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>8. Given a linear relation, constructs a table of values and a graph for that relation.</p>	<p>Give an example of a linear relation such as Journeys in Math 8 TRM, p. 310.</p> <p>Construct a table of values, generate the ordered pairs, and graph the relation.</p> <p>At this time discuss the merits of a graph as opposed to a table of values.</p> <p>e.g.,</p> <ul style="list-style-type: none"> – visible and more understand-able representation of a relationship. – can be used to predict or hypothesize. 	<p>(E) Given a linear relation of the form $y = ax + b$, explore how changing one aspect (a or b) of the relation changes the graph.</p> <p>(E) Have students write relations and exchange these in the class for evaluating and graphing.</p> <p>(R) Give a table of values for students to construct a graph on grid paper.</p> <p>Given a linear relation of the form $y = ax + b$, construct a table of values.</p> <p>Using substitution with given x values, students find y.</p> <p>Extending this, students generate their own values for x to find y.</p> <p>From a table of values students try to determine the defining relation by looking at the pattern.</p> <p>Explore number patterns that arise from tables of values.</p> <p>This may open discussion about choosing x values, choice of scale, and extension to more than one quadrant.</p>	<p>Houghton Mifflin Mathematics 8, "Activities Courseware Solutions: One Equation", p. 67.</p> <p>Looking for a pattern is one problem-solving technique.</p> <p>Collecting and organizing information in charts and graphs is a problem-solving skill.</p>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
8. Given a linear relation, constructs a table of values and a graph for that relation. (cont'd)	By graphing, for example $y = 2x + 3$, and $y = 2x + 5$ on the same axis, students explore how changing one part of the relation changes the graph.		

GRADE 9
MATHEMATICS



PLEASE NOTE

The following resources have received basic status and are available from the Learning Resources Distributing Centre:

Student texts for JOURNEYS IN MATH 7, 8

Student texts for HOLT MATH 7, 8, 9

Any other titles mentioned in this guide have not received formal status approval. They are presently under review and cannot, at this time, be obtained through the Learning Resources Distributing Centre.

CURRICULUM GUIDE (DRAFT)

PROBLEM SOLVING

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>1. Demonstrates an understanding of a problem-solving situation.</p> <p>2. Demonstrates a willingness to find a solution to a problem.</p> <p>3. Uses a variety of strategies to solve problems. Previously developed strategies are used.</p> <p>The following strategies should be developed throughout the various strands of the program and within the problem-solving framework:</p> <ul style="list-style-type: none"> a) Understanding the problem ● considers alternative interpretations ● makes assumptions 	<p>Refer to introductory comments under Objective #1 in Grade 7.</p> <p>Refer to introductory comments under Objective #2 in Grade 7.</p> <p>As students encounter more complex problems, the skills required to solve them become more intellectually complex. Consequently, students must utilize higher-level thinking skills such as logic or reasoning.</p> <p>As a teacher demonstration involving the class, the strategy of using logic and reason can be developed within the problem-solving framework (Understanding the problem, Developing a plan, Carrying out the plan, Looking back) as follows:</p> <p>Margie is a blonde, Rose Mary a redhead, and Shirley is a brunette. They are married to Alex, Frank, and John but</p> <ul style="list-style-type: none"> a) Shirley does not like John. 	<p>Teachers must recognize that problem-solving skills are essential for all students and that being perplexed, when first encountering a problem, is normal. Problems presented to students should be challenging yet solutions must be attainable to insure that students experience success. It is very important for teachers to realize individual student differences in learning; therefore the growth expectation should also vary. Students who experience difficulty with the complex strategies may find it necessary to use a more concrete approach for a longer period of time and may require more teacher guidance.</p> <p>a) The use of calculators in problem solving must be encouraged so that time spent on tedious calculations is decreased and feedback on strategies is faster. Numbers from realistic and relevant situations are less imposing if calculators are used.</p> <p>b) Group work should often be used in problem solving. A student in a group deals with ideas and questions from other members of the group, and this may help each student to progress in developing problem-solving strategies.</p>	

Note: (E) = Enrichment
(R) = Remediation

PROBLEM SOLVING

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>3. b) Developing a plan (choosing a strategy)</p> <ul style="list-style-type: none"> ● formulates an equation ● uses logic or reason ● constructs flow charts ● develops a symbol or code system ● recognizes limits and eliminates possibilities 	<p>b) Rose Mary is married to John's brother.</p> <p>c) Alex is married to Rose Mary's sister.</p> <p>Who is married to whom? Assume that married people like each other! (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985 p. 40, #17.)</p> <p>Under the guidance of the teacher, students investigate the strategy by solving a similar but non-related problem such as:</p> <p>There are eight baseballs, all exactly alike in size and appearance, but one is heavier than any of the other seven which are all the same weight. With a balance scale, how can the heaviest baseball be positively determined with only two weighings? (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, p. 35, #9.9.)</p> <p>The natural progression leads to students using the same strategy to solve problems such as:</p> <p>a) A ferryboat, when filled, can carry 6 Pintos and 7 Toyotas or 8 Toyotas only, then what is the maximum number that it can carry? (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, p. 42, #39.)</p>	<p>A teacher should challenge the more capable students by having them not only justify their strategies and solutions but also to consider other possibilities such as:</p> <ul style="list-style-type: none"> ● other strategies and solutions ● "what if?" (change an element of the problem) ● generalization of rules to other situations. 	<p>c) Computers may be used to assist in teaching problem solving. Various programs and simulations require the use of particular or various strategies.</p> <p>d) The use of relevant and realistic problems (from local sources such as newspapers and magazines) is encouraged because this will increase the interest of the students.</p> <p>Students may contribute their own ideas of problems; in addition to collecting data they may make up questions related to the information. An exchange of problems and questions may be encouraged.</p> <p>Examples can be obtained from such sources as: Mathematics Activity Software (MAC) by Houghton Mifflin, Minnesota Educational Computing Consortium (available through ACCESS) and Sunburst Communications Software.</p>

PROBLEM SOLVING

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>d) Looking back (cont'd)</p> <ul style="list-style-type: none"> • generalizes solutions • creates and writes routine and non-routine problems. 	<p>b) Bill, John, Joe, and Henry have to catch the six o'clock bus.</p> <ol style="list-style-type: none"> i) Bill's watch is 10 minutes fast, but he thinks it is 5 minutes slow. ii) John's watch is 10 minutes slow, but he thinks it is 10 minutes fast. iii) Joe's watch is 5 minutes slow, but he thinks it is 10 minutes fast. iv) Henry's watch is 5 minutes fast, but he is under the impression it is 10 minutes slow. <p>If each leaves to catch the bus he will just make it, if his time is what he thinks it is. Who misses the bus? (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, p. 44, #5.)</p>	<p>Evaluation</p> <p>To evaluate problem solving requires more than grading the solution to mathematical problems. Continual observation and questioning of students while they are solving problems is essential.</p> <ul style="list-style-type: none"> • willingness to attempt problems • use of systematic approach • selection of appropriate strategies 	

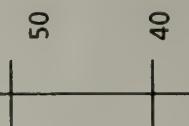
PROBLEM SOLVING

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>d) Looking back (cont'd)</p> <ul style="list-style-type: none"> • generalizes solutions • creates and writes routine and non-routine problems. 	<ul style="list-style-type: none"> ● efficiency in selection of appropriate strategies ● logical justification of strategies and solutions ● growth in confidence in problem-solving ability ● transfer of problem-solving skills to situations other than mathematics. <p>(Evaluation techniques and instruments for problem solving are found in the Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, pp. 7, 8, 52-56.)</p>		

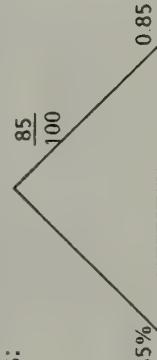
NUMBER SYSTEMS AND OPERATIONS

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>1. Applies and practises problem-solving skills in new situations.</p> <p>2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations.</p>	<p>An equal emphasis should be placed on the various strategies for computing. Single-digit basic facts should be drilled on a regular basis through activities such as timed challenges or games. Paper-and-pencil strategies should be used to develop an understanding of sub-concepts such as re-grouping, borrowing or place value. Long and tiresome paper-and-pencil drill is discouraged.</p> <p>Estimation should be done on a daily basis. Recognition of appropriate situations for estimates, determining how precise an estimate should be for a given situation and knowing when a computed answer is possible are among skills to be emphasized.</p>	<p>Example:</p> <p>Range: </p> <p>$15 + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$ 40</p>	<p>A good calculator estimation activity is the "Range Game". <u>Estimation and Mental Computation</u>. Reston, VA: National Council of Teachers of Mathematics, 1986, pp. 182-185.</p> <p>"<u>The Range Game</u>"</p> <p>"The Range Game" builds estimation skills. The only tools needed are a calculator and paper and pencil for record keeping. The object is to find numbers within a given range that will satisfy a given equation. Introduce the game by displaying a range and a partial equation, as shown below.</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations. (cont'd)	<p>Ask questions such as the following:</p> <ul style="list-style-type: none"> ● "Find a number that when added to 15 gives a sum in the range shown." ● "What is the largest number that works?" "The smallest?" ● "Are there any other numbers?" ● "Let's find all the numbers that will work." 	<p>List responses on the board and discuss the findings. Ask, "How many numbers work?"</p>	<p>Suggest calculator problems such as those from NCTM "Activities for Junior High School and Middle School Mathematics", p. 202. Play a game called "Erase". Have your friend enter any six-digit number into the calculator. Now see what is the fewest number of moves it takes you to get to a display of zero. For each move, you may add, subtract, multiply, or divide by any two-digit, non-zero number.</p>
3. Maintains previously developed skills with whole numbers, integers, decimals and fractions (operations, ordering, relationships among systems, need for rational numbers, order of operations).	<p>Use time drills with basic number facts, in particular multiplication and addition. Encourage estimation, mental computation and the use of a calculator.</p> <p>e.g., An activity that demonstrates a relationship among number systems is:</p> $\frac{85}{100} = 0.85$	<p>(R) Rather than doing the actual division ask students to write the number of digits in the whole number part of the quotient.</p> <p>e.g., $0.98 \overline{)10.03}$ 2 digits</p>	

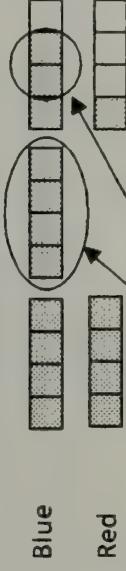
NUMBER SYSTEMS AND OPERATIONS

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3 Maintains previously developed skills with whole numbers, integers, decimals and fractions (operations, ordering, relationships among systems, need for rational numbers, order of operations). (cont'd)	<p>TRIPLE CHECK</p> <p>Usually by the first of March, my sixth-grade pupils have demonstrated that given a rational number in any of the three forms, fraction, decimal, or percent, they can quickly give the two equivalent names for commonly used fractions and can find equivalents for other fractions, decimals, or percents.</p>	<p>(R) Ask students to place decimal points in four different ways to make four true number sentences in each set.</p> <p>e.g., $124 \times 62 = 7688$ $124 \times 62 = 7688$ $124 \times 62 = 7688$ $124 \times 62 = 7688$</p> <p>(Idea from <i>Arithmetic Teacher</i>, Vol. 34, #7, March 1987.)</p> <p>(E) Investigate the stock market, 'Why are fractions used rather than decimals?'</p>	<p>Trade places and have your friend try to "erase" a six-digit number you enter into the calculator.</p> <p>What did you notice about numbers that make them easy or difficult to erase?</p> <p>(From NCTM "Activities for Junior High and Middle School Mathematics", p. 202.)</p> <p>To maintain and increase that knowledge, we use Triple Check at various intervals throughout the rest of the school year. On the last day of each month, pupils give their daily attendance, reading and mathematics grades, and spelling scores in "Triple Check". Usually we begin with a fraction, for example, for 17 days of attendance of a possible 20 days, we would write $\frac{17}{20} = \frac{85}{100} = 85\% = 0.85$. However, if you are working with decimals or percents, name them first.</p> <p>Triple Check encourages pupils to check their answers in a reasonable and an exact manner. It reinforces the idea that fractions, decimals, and percents communicate the same information about rational numbers.</p> <p>(From the file of Celestine Wyatt, 10629 S. Emerald, Chicago, IL 60628. <i>Arithmetic Teacher</i>.)</p>

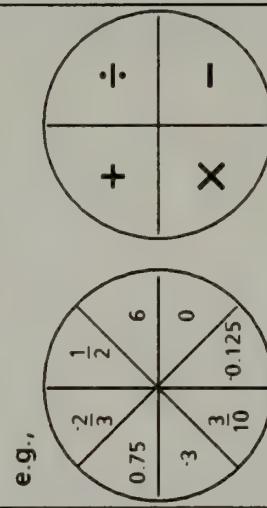
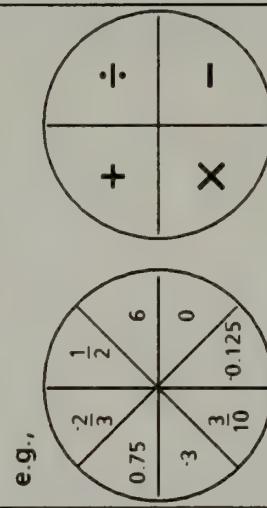
NUMBER SYSTEMS AND OPERATIONS

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>4. Performs the operations of addition, subtraction, multiplication and division with rational numbers.</p>	<p>Review the operations with positive rational numbers (see Grade 8 Objective #4) and the operations with integers (see Grade 8 Objective #3).</p> <p>Concrete activity: $-2\frac{3}{4} + 1\frac{1}{4}$</p> <p>Use teacher-made manipulatives consisting of rectangular shapes divided into quarters. (Use overhead.)</p>	<p>Blue  (negative rational)</p> <p>Red  (positive rational)</p>	<p>To find the answer to the above problem balance as many complete boxes and quarters of boxes as possible.</p> <p>e.g., 1 red box balances 1 blue box. $\frac{1}{4}$ red box balances $\frac{1}{4}$ blue box.</p>  <p>This leaves 1 blue box and $\frac{2}{4}$ blue box or $-1\frac{2}{4} = -1\frac{1}{2}$ as the result.</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
5. Applies the rules for order of operations to evaluate expressions involving rational numbers in any of their forms.	<p>Review the rules for order of operations with whole numbers, fractions and integers. Extend the concepts to rational numbers.</p> <p>(R) Working in groups of two, develop spin game. Teacher instructs how many spins of each. A problem involving order of operations emerges.</p> <p>Have students estimate and then calculate their answers. A discussion will follow focusing on order of operations rules.</p> <p>e.g.,</p> 	<p>(R) Working in groups of two, develop spin game. Teacher instructs how many spins of each. A problem involving order of operations emerges.</p> <p>Have students estimate and then calculate their answers. A discussion will follow focusing on order of operations rules.</p> <p>e.g.,</p> 	<p>Explore the operating system of a calculator by entering an operation in sequence. Ask the students to explain the operating system of their own calculator (e.g., Is the order of operation "built in"?).</p> <p>Encourage the use of calculators.</p>

NUMBER SYSTEMS AND OPERATIONS

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>6. Converts rational numbers from $\frac{a}{b}$ form to decimal form (limit: $b < 10$ or b is a power of 10).</p>	<p>Students have converted rational numbers in the form $\frac{a}{b}$ to decimal form by converting to denominators of 10, 100, or 1000 and by using a calculator.</p> <p>Students should now be introduced to the paper-and-pencil algorithm of dividing the numerator by the denominator.</p>	<p>(E) Explore the resulting decimal forms of different rationals ($\frac{a}{b}$) – terminating, non-terminating, repeating or non-repeating. In particular an interesting activity is to contrast fractions with denominators of 11 to fractions with denominators of 9.</p> <p>(E) Post this activity on the bulletin board but let students determine the answers.</p> <p>Stress should be placed on the meaning of the symbol “–” i.e., means 3 out of 4, three quarters and 3 divided by 4.</p> <p>Emphasis should be placed on the following:</p> <ol style="list-style-type: none"> meaning of symbols \div and $\frac{a}{b}$ (bar) numbers can have different forms (decimal, fraction). 	<p>Calculate the digits in the decimal equivalents of common fractions using the following BASIC program for division.</p> <pre> 5 REM Decimal equivalents 10 HOME 20 PRINT "This program will find decimal equivalents to 5 and 8 decimal places." 30 PRINT 40 INPUT "What is the numerator?";D 50 INPUT "What is the denominator?";D 60 F = N/D 70 F5 = INT(F*(10^5) + .5)/ INT(10^5) 80 F8 = INT(F*(10^8) + .5)/ INT(10^8) 90 PRINT 100 PRINT "The fraction, "N;"1";D;" is equal to:" 110 PRINT F5; "(to 5 places)" 120 PRINT F8; "(to 8 places)" 130 END </pre> <p>(E) Explore non-terminating decimals (irrational numbers) e.g., “π” in Journeys in Math 9 TRM, p. 81.</p>

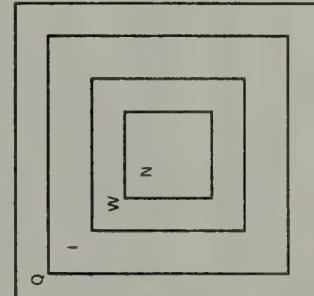
NUMBER SYSTEMS AND OPERATIONS

Grade 9

	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology																				
6. Converts rational numbers from $\frac{a}{b}$ form to decimal form (limit: $b < 10$ or b is a power of 10). (cont'd)	<p>(E) Change repeating decimals to fractions. See Holtmath 9 Teacher's Edition, pp. 76-77.</p> <p>(R) 4-in-a-Line</p> <p>Rules</p> <ul style="list-style-type: none"> Take turns. Pick any two of these numbers to make an $\frac{a}{b}$ fraction. Divide. Mark the decimal name for your fraction on the game board (use x or o). Four marks in a line wins <table border="1" data-bbox="651 665 713 940"> <tr> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>8</td> </tr> </table> <p>Game Board</p> <table border="1" data-bbox="870 602 1059 1004"> <tr> <td>1.3</td> <td>0.6</td> <td>0.625</td> <td>1.25</td> </tr> <tr> <td>1.6</td> <td>0.375</td> <td>0.83</td> <td>0.75</td> </tr> <tr> <td>0.8</td> <td>1.2</td> <td>0.6</td> <td>1.5</td> </tr> <tr> <td>1.6</td> <td>2.6</td> <td>0.5</td> <td>2.0</td> </tr> </table> <p>NCTM - Feb. 1984.</p>	3	4	5	6	8	1.3	0.6	0.625	1.25	1.6	0.375	0.83	0.75	0.8	1.2	0.6	1.5	1.6	2.6	0.5	2.0	<p>(Holtmath 9 Teacher's Edition, pp. 76-77.)</p> <p>Encourage mental computation and check answers using calculators.</p> <p>7. Converts rational numbers from decimal form to $\frac{a}{b}$ form (limit: terminating decimals).</p> <p>Can use a calculator to explore patterns. (See Journeys in Math 7 TRM, p. 191.)</p>
3	4	5	6	8																			
1.3	0.6	0.625	1.25																				
1.6	0.375	0.83	0.75																				
0.8	1.2	0.6	1.5																				
1.6	2.6	0.5	2.0																				

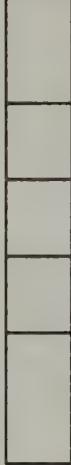
NUMBER SYSTEMS AND OPERATIONS

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology																																			
8. Computes the square root of whole numbers using estimation and a calculator.	<p>Encourage students to estimate before calculating.</p> <p>Start with the idea of squaring. 144 means 12×12. Then go into the opposite operation of square roots.</p> <p>i.e., $8 \times 8 = 64$ $\therefore \sqrt{64}$ is 8</p> <p>Establish a table to show squaring and square root are opposite operations.</p> <p>See Holtmath 9 Teacher's Edition, p. 174.</p>	<p>(R) Use prime factorization to find square roots.</p> $\sqrt{576} = 24$ $576 = (2 \times 2 \times 2 \times 3) (2 \times 2 \times 3)$ $(24) \quad (24)$ <p>$\begin{array}{r} 2 \longdiv{576} \\ 2 \longdiv{288} \\ 2 \longdiv{144} \\ 2 \longdiv{72} \\ 2 \longdiv{36} \\ 2 \longdiv{18} \\ 3 \longdiv{9} \\ 3 \longdiv{3} \end{array}$</p> <p>See Holtmath 9 Teacher's Edition, p. 176.</p>	<p>MAC 8, Disk B, "Square Root Chase Game" reviews estimation, squares and square roots.</p>																																			
9. Demonstrates the relationship among whole numbers, integers and rational numbers.	<p>The intent is for students to demonstrate the relationship between the sets. One way this can be demonstrated is pictorially.</p> 	<p>Have students complete a chart.</p> <table border="1" data-bbox="1035 591 1380 1014"> <tr> <td></td> <td>N</td> <td>W</td> <td>I</td> <td>Q</td> </tr> <tr> <td>1</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>$\sqrt{3}$</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>$\frac{3}{5}$</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>-0.125</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>π</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>3</td> <td></td> <td></td> <td></td> <td></td> </tr> </table>		N	W	I	Q	1					$\sqrt{3}$					$\frac{3}{5}$					-0.125					π					3					<p>See Holtmath 9 Teacher's Edition, p. 176.</p>
	N	W	I	Q																																		
1																																						
$\sqrt{3}$																																						
$\frac{3}{5}$																																						
-0.125																																						
π																																						
3																																						

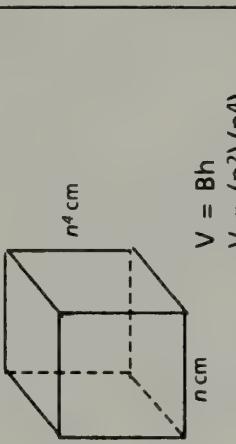
NUMBER SYSTEMS AND OPERATIONS

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
9. Demonstrates the relationship among whole numbers, integers and rational numbers. (cont'd)	<p>Use a fraction tape to illustrate the relationship.</p> 		
10. Understands and uses the following properties (limit: numerical bases):	<p>For a concrete approach refer to the Grade 7 and 8 Number systems and Operations sections of this curriculum guide.</p> <p>Have students explore and generate exponent laws using numbers.</p> <p>e.g.,</p> <ul style="list-style-type: none"> 1. $5^2 \times 5^3 = 5^{2+3}$ 2. $5^3 = 125$ 3. $5^1 = 5$ 4. $5^0 = 1$ 5. $5^{-1} = \frac{1}{5^1}$ 6. $5^{-2} = \frac{1}{5^2}$ <p>3. $5^5 \div 5^3 = 5^{5-3}$</p> <p>4. $(5^2)^3 = 5^{(2)(3)}$</p>		

NUMBER SYSTEMS AND OPERATIONS

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>Extended Content</p> <p>Understands and uses the above properties from Objective #10 as literal bases.</p>	<p>Have students explore and generate exponent laws using literal bases.</p> <p>e.g., 1. $(x^2)(x^3) = (x)^{2+3}$</p> <p>2. $(x^{-2}) \div (x^{-3}) = \left(\frac{1}{x^2}\right) \left(\frac{1}{x^3}\right)$</p> $= \frac{1}{x^{2+3}}$ $= \frac{1}{x^5}$	<p>(E) Find area and volume.</p> <p>e.g.,</p>  $V = Bh$ $V = (n^2)(n^4)$ $V = n^6 \text{ cubic cm}$	<p>(R) Multiply by powers of 10 (positive and negative exponents).</p> <p>(See Holtmath 9 Teacher's Edition, p. 170.)</p> <p>At the Grade 9 level, this could be extended to negative exponents.</p> <p>e.g., $0.000000002563 = 2.563 \times 10^{-12}$</p>

NUMBER SYSTEMS AND OPERATIONS

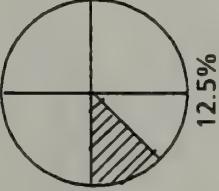
Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>Extended Content</p> <p>Computes with numbers written in scientific notation (multiplying and dividing).</p>	<p>Apply exponent laws to computing numbers in scientific notation.</p> <p>e.g., $\begin{aligned} & (8.7 \times 10^{12}) \times (3.3 \times 10^{-8}) \\ & = (8.7 \times 3.3) \times (10^{12} \times 10^{-8}) \\ & = 28.71 \times 10^4 \\ & = 2.871 \times 10^5 \end{aligned}$</p> <p>Start with computing large numbers, i.e., $6\,823\,000 \times 7\,433\,000$.</p> <p>Is there a better way of handling this? Can lead to calculators and computers in handling scientific notation.</p>	<p>(E) Develop a method for adding or subtracting numbers in scientific notation.</p> <p>e.g., 1. $\begin{aligned} & 5 \times 10^5 + 6 \times 10^3 \\ & = 5.06 \times 10^5 \end{aligned}$</p> <p>2. $\begin{aligned} & 8 \times 10^8 + 5 \times 10^1 \\ & = 8.000\,000\,5 \times 10^8 \end{aligned}$</p> <p>i.e., $3.43501\text{E} + 05$.</p>	<p>Using calculators and computers, multiply large numbers and observe the results when the instruments express the answers in scientific notation.</p>

RATIO AND PROPORTION

Grade 9

Objective	Clarification or Example	Elective Suggestions Integration of Problem Solving and Technology
<p>1. Applies and practises problem-solving skills in new situations.</p> <p>2. Maintains previously developed skills (understands and constructs ratios, rates and proportions; finds the missing element of a proportion; writes ratios as percents; converts fractions and decimals to percents and percents to fraction and decimal forms; finds missing values in commission, sales tax, and discount situations).</p> <p>3. Converts fractional percents to fraction and decimal forms:</p> <p>e.g., $12\frac{1}{2}\% = \frac{1}{8} = 0.125$</p>	<p>Holtmath 7 Software Disk 4.</p>	<p>(R) Encourage mental computation with simple fractional percentages.</p> <p>Review equivalent forms: ratios, percents, decimals (limit: whole no. %'s).</p>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>3. Converts fractional percents to fraction and decimal forms: (cont'd)</p> <p>e.g., $12\frac{1}{2}\% = \frac{1}{8} = 0.125$</p>	<p>Develop the concept of fractional percentages at a semi-concrete level before formal development occurs.</p> <p>e.g.,</p>  <p>12.5% 100% = $\frac{8}{8}$ (1) $87\frac{1}{2}\% = \frac{7}{8}$ 75% = $\frac{6}{8}$ ($\frac{3}{4}$) $62\frac{1}{2}\% = \frac{5}{8}$ 50% = $\frac{4}{8}$ ($\frac{1}{2}$)</p> <p>Use flash cards for drill and practice.</p> <p>Encourage the use of calculators.</p>	<p>Have students look for patterns: e.g.,</p>	<p><u>Problem-Solving Project</u> Investigate the cost of purchasing a car vs renting a car. e.g., payments interest insurance maintenance gas</p> <p>(E) Have students determine monthly payments for a car loan. Provide students with information as to: down payment required; current interest rate; length of payment period.</p>
<p>4. Finds any one of the missing elements (value or percent) in applications such as simple interest, commission, sales tax, discount, profit and loss, and percent increase and decrease situations.</p>	<p>This objective is an extension of Objective #5 (Ratio and Proportion, Grade 8) where students were always given the percent and asked to find a missing element. Finding the percent (given other elements) is subtly different. Solutions will occur in fraction or decimal form and must be converted to percent.</p>		

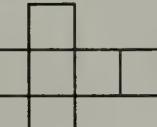
RATIO AND PROPORTION

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>4. Finds any one of the missing elements (value or percent) in applications such as simple interest, commission, sales tax, discount, profit and loss, and percent increase and decrease situations. (cont'd)</p>	<p>(E) Have students investigate the difference in total amount of money payable if a house mortgage is amortized over 15 years, 25 years and 35 years. Amortization tables are required.</p> <p>(E) With the use of calculators, have students calculate and compare the amount of interest they would earn if a given sum of money was placed in a bank account where interest was compounded semi-annually or monthly.</p>	<p>Compound interest is not always calculated annually. It could be calculated semi-annually, quarterly, monthly, or daily. Find a formula to calculate interest compounded "n" times during a year. (From Holtmath 9 Teacher's Edition, p. 209.)</p> <p>Discuss different bank accounts and the advantage of one over another.</p> <p>i.e., DICA Regular savings (interest twice a year) Daily interest.</p>	
<p>5. Interprets maps and scale drawings.</p>	<p>Use a class set of atlases to generate a discussion about scale:</p> <ol style="list-style-type: none"> What is it? How is it expressed? Calculate actual and scale distances. <p>Discuss trips students have taken and use maps to get the actual from the scale distance.</p>	<p>(E) Students are planning a trip in which they rent a car. They want to calculate costs for distance using the scale drawing of a map.</p> <p>(R) Review of metric conversions cm to km, cm to m.</p>	<p style="text-align: right;">- 152 -</p>

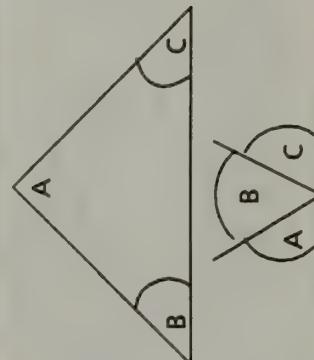
RATIO AND PROPORTION

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>6. Uses a scale to construct drawings, maps or pictures.</p>	<p>Ask students to draw a cartoon character, a company logo, a rock band logo, a map or a plan of a building, room, etc., on a smaller (or larger) scale.</p> <p>Students should be taught how to enlarge or reduce with a scale.</p> <p>The criteria for evaluating the project should be clearly defined (e.g., accuracy of scale, proportion, neatness).</p>	<p>Students who experience difficulty with this should be encouraged to use grid paper to help them with proportion.</p> <p>(E) Give students two drawings, actual and scale. Have them work out the key.</p> <p>(E) Have the students measure and draw a scale diagram of the classroom.</p>	<p>Have a student use scale drawings in problems such as:</p> <p>Merv has a garden plot 12 cm by 9 m. He wishes to plant vegetable plots 2 m by 9 m. What is the maximum number of plots Merv can plant? How many arrangements are possible?</p> <p>Have student investigate problems to determine when the most economical buy is not always the best buy.</p> <p>(E) Give students their marks for the term with each of the assigned weightings. Have them calculate their term marks. Compare the results to the computer printout.</p> <p>(R) Have students check unit prices in their local grocery store.</p> <p>(R) Determine which item is the better buy by finding the unit prices.</p> <p>e.g., school logo cars mascot</p> <p>From this 'net' drawing construct a model which is 6 times as large (1:6).</p>  <p>e.g., 3 blank tapes for \$5.99 or 2 blank tapes for \$4.69.</p>
<p>7. Applies ratio and proportion in practical situations (e.g., uses shadows to find the height of a pole or building; comparative shopping; building a model; computing a test or report card mark based on weighted averages).</p>			

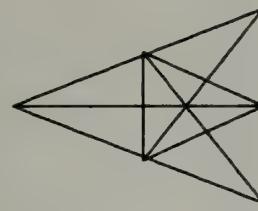
MEASUREMENT AND GEOMETRY

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>1. Applies and practises problem-solving skills in new situations.</p> <p>2. Maintains previously developed skills (linear, area, volume, capacity, and mass units of measure; classification of polygons; perimeter and area of polygons and the circle; volume of a right rectangular prism and cube).</p> <p>3. Uses concrete manipulatives to determine the sum of the angles in a triangle (180°).</p>	<p>Problem: You have a 5 L jar and a 3 L jar, both of which are not marked in any way. You may use as much water as you need. Describe how to obtain exactly 4 L of water by filling and emptying the jars.</p> <p>Remediation: Have students measure and record angles of various triangles, then determine the sum for each.</p> <p>This exercise is an appropriate point to discuss accuracy of measurements.</p> <p>Draw and cut out any triangle. Then cut or tear each angle from the triangle. Place them together to show that the sum of the interior angles equals 180°.</p> 		

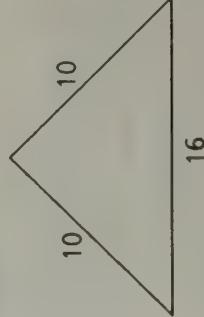
MEASUREMENT AND GEOMETRY

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>4. Determines the sum of the interior angles in polygons.</p> <p>Explore various strategies that could be used to determine the sum of angles in any given polygon. One possibility is by measuring each angle with a protractor and then determining the sum. Another strategy is to divide the shape into triangles as shown.</p> <p>e.g.,</p>		<p>square = 2 triangles $= 2 \times 180^\circ$ $= 360^\circ$</p> <p>•• sum of interior angles is 360°</p> <p>e.g.,</p>	<p>pentagon = 3 triangles $= 3 \times \text{-----}$ $= \text{-----}$</p> <p>•• sum of interior angles is -----</p> <p>Problem- Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, p. 47.</p> <p>Problem: How many triangles can you count?</p> 

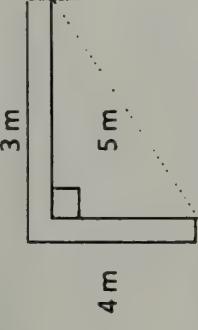
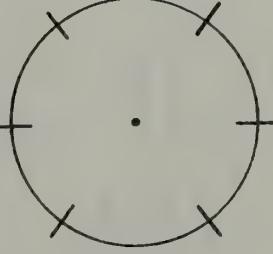
MEASUREMENT AND GEOMETRY

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>5. Uses concrete manipulatives to develop the Pythagorean relationship in right triangles.</p> <p>(See Holtmath 9 Teacher's Edition, p. 179, "Alternative Teaching Strategies".)</p>	<p>The Pythagorean theorem can be used to find the distance between two points on a grid.</p> <p>(E) Research the historical development of the Pythagorean theorem.</p>	<p>(E) Explore Pythagorean triplets. A possible approach is to have students write a computer program to generate Pythagorean triplets.</p> <p>Use a calculator to generate Pythagorean triplets using the expressions:</p> $(m^2 - n^2, 2mn, m^2 + n^2)$ <p>(E) Explore the contention that the products of Pythagorean triplets are always divisible by 60. Does it work? Why or why not?</p>	<p>Problem</p> <p>a) An isosceles triangle with sides of length 10 cm and 16 cm is folded in half. How long is the crease?</p> 
<p>6. Applies the Pythagorean relationship to practical situations.</p>	<p>The Pythagorean theorem lends itself to solving many practical problems. Some strategies to reinforce are drawing a diagram, making a simple problem, developing a code or symbol system, documenting the process, looking back to determine if the solution is reasonable, making and solving other similar problems.</p>	<p>(E) The Pythagorean theorem can be used to find the distance between two points on a grid.</p>	

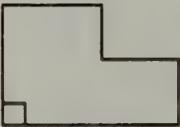
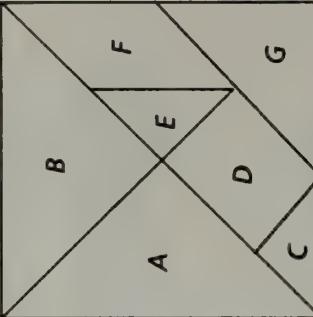
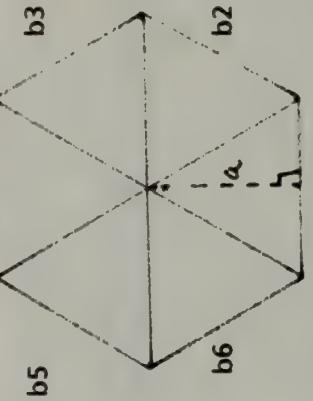
MEASUREMENT AND GEOMETRY

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>6. Applies the Pythagorean relationship to practical situations. (cont'd)</p> <p>e.g., A carpenter wishes to build a square corner in framing a basement. He measures distances of 3 m and 4 m on the 2×4's on the floor. The distance of 5 m ensures a square corner.</p>		<p>After discussing regular polygons, work through a construction.</p> <p>e.g., an inscribed hexagon.</p> <p>7. Constructs regular polygons using tools such as a computer, ruler, protractor and/or compass.</p>	<p>Use a graphics or LOGO program to generate regular polygons. Investigate their attributes:</p> <p>(a) measure of angles (b) measure of sides (c) relationship of perimeter to diagonal (diameter)</p> <p>(E) Have students create designs using polygons constructed.</p> <p></p> <p>- Why does it work? - How can we use this to get an equivalent triangle? - How can we get an inscribed square? Octagon?</p>

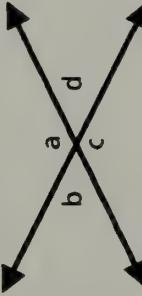
MEASUREMENT AND GEOMETRY

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>8. Understands and uses a strategy to determine the area of a regular polygon.</p>	<p>Build on previous skills (area of triangles, parallelograms, and trapezoids). Expand strategies to find areas of many polygons. For example, a hexagon can be divided into 6 triangles.</p> <p>b4</p>	<p>(E) Expand basic strategies to finding areas of compound figures.</p> <p>e.g.,</p>  <p>The area of this can be found by dividing the figure.</p> <p>(R) Use cutout pieces to make shapes.</p> <p>(Arithmetic Teacher, Vol. 31, #5, January 1984, pp. 28-32.)</p>	<p>*The figure shows a square separated into the seven pieces of an ancient puzzle called the tangram. If the area of the entire square is one square unit, what is the area of each of the seven tangram pieces?</p>  <p>*Creative Problem Solving in School Mathematics, Houghton Mifflin, p. 134.</p> <p>b5</p>  <p>b1</p> <p>Total area = sum of 6 triangles.</p> $A = \frac{1}{2}a(b_1 + b_2 + \dots + b_6)$ $A = \frac{1}{2}ap$ $A = \frac{1}{2}(\text{apothem})(\text{perimeter})$ <p>Note:</p> <p>$P = ns$ (perimeter = no. of sides + length of side)</p>

MEASUREMENT AND GEOMETRY

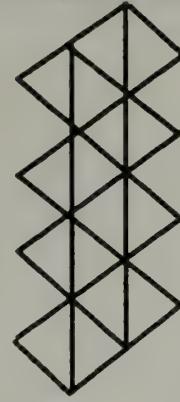
Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>9. Identifies pairs of angles: (supplementary, complementary, adjacent and opposite).</p>	<p>This is the appropriate time to reinforce the use of a ruler and a protractor. Have students draw two intersecting lines and then measure the resulting angles. They will then discover the resulting relationships.</p>	 <p>Angles:</p> $ \begin{aligned} a + b &= 180^\circ \\ b + c &= 180^\circ \\ c + d &= 180^\circ \\ d + a &= 180^\circ \\ a = c \\ b = d \end{aligned} $	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>10. Uses a compass and a straightedge to construct:</p> <ul style="list-style-type: none"> ● a congruent segment ● a congruent angle ● a perpendicular bisector of a segment ● a bisector of an angle ● a perpendicular to a line ● angles of 90°, 45°, 60°, and 30°. 	<p>(Holtmath 9 Teacher's Edition, pp. 220-229.)</p> <p>Angle construction is a direct extension of the following angles:</p> <ul style="list-style-type: none"> 90°— a perpendicular 45°— bisector of a 90° angle 60°— construction of an equilateral triangle 30°— bisector of a 60° angle <p>Given a net of a right rectangular prism, have students fold and paste to make a box. Explore its construction making note of its characteristics (faces, edges, vertices). Expand this to students exploring other right prisms (for example, a cube, a triangle prism, a pentagonal prism) and their construction.</p> <p>11. Given nets, constructs right prisms.</p> <p>(E) Explore other constructions, such as constructing a perpendicular to a point not on a line, finding an altitude of a triangle.</p> <p>(E) Show students examples of other figures (tetrahedron, dodecahedron, and so on). Explore the nets for these.</p> <p>(R) Have a collection of boxes (cracker boxes, cereal boxes, matchboxes, etc.). Students can then dismantle the boxes and examine the nets. Encourage them to look for the similarities and differences.</p> <p>Problem Solving: Discuss how to produce nets of right prisms or cylinders, given specific conditions. e.g.,</p> <p>You have a piece of paper (loose-leaf size). Construct a net for a cylinder that has a 5 cm radius. Problems of this type have more than one solution since the condition net specified is the height.</p>		

MEASUREMENT AND GEOMETRY

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology																												
<p>11. Given nets, constructs right prisms. (cont'd)</p> <p>Extended Content</p> <p>Given nets, constructs right pyramids.</p>		<p>Problem:</p> <table border="1" data-bbox="373 587 577 1031"> <tr> <td>1</td><td>2</td><td></td><td></td><td>4</td><td></td><td></td></tr> <tr> <td></td><td>3</td><td>2</td><td></td><td>6</td><td>3</td><td></td></tr> <tr> <td>4</td><td></td><td>5</td><td>3</td><td>1</td><td></td><td>2</td></tr> <tr> <td></td><td>5</td><td>6</td><td>4</td><td></td><td></td><td>1</td></tr> </table> <p>Each of the above patterns can be folded to form a cube. Which two cubes will look the same?</p> <p>(E) String art on an octahedron constructed of drinking straws. (Arithmetic Teacher, Vol. 34, #3, November 1986, pp. 30-33.)</p>	1	2			4				3	2		6	3		4		5	3	1		2		5	6	4			1	<p>* Have a three-dimensional model of an icosahedron available for students to examine. Give everyone a pattern for constructing an icosahedron. Ask each student to colour a pattern such that, after the figure has been cut out and taped together, no two adjacent faces (faces that share an edge) will be the same colour. Furthermore, everyone should attempt to use the fewest number of colours. Finally, have the students cut out their patterns and tape them together to check their solutions.</p> <p>(diagram)</p>  <p>* (Arithmetic Teacher, Vol. 31, #7, March 1984, p. 43.)</p>
1	2			4																											
	3	2		6	3																										
4		5	3	1		2																									
	5	6	4			1																									

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
12. Classifies right prisms and cylinders.	<p>From exploration of constructing nets, students will have an intuition as to the characteristics of various right prisms and cylinders. Classification is simply a natural progression.</p> <p>i.e., rectangular right prism triangular right prism square right prism (cube) right circular cylinder</p>	<p>Extended Content</p> <p>Classifies right pyramids.</p>	<p>Problem Solving: Strategies can be developed and expanded upon through problem solving. For example "Susan built a toy chest for her little brother. The dimensions of the toy chest are 95 cm by 40 cm by 50 cm. Susan wanted to paint the outside (including the bottom) with enamel paint. What is the total surface to be painted?"</p>
13. Understands and uses a strategy for finding the surface area of any right prism or cylinder.	<p>The development of a strategy for finding the surface area of right prisms and cylinders can be easily approached by building on students' investigation of nets. Some students may divide prisms into faces, while others may decide to develop a formula.</p> <p>i.e.,</p>	<p>(R) Using models of prisms and cylinders, take them apart and explore the area of each piece.</p>	<p>A triangular prism is made up of 3 rectangles and 2 triangles.</p>

MEASUREMENT AND GEOMETRY

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>14. Understands and uses a strategy for finding the volume of any right prism or cylinder.</p> <p>Students, in previous work, have used a formula for finding the volume of a cube or right rectangular prism. Using blocks, build a right rectangular prism.</p> <p>By dividing the prism into two congruent triangular prisms, a strategy for determining the volume of a triangular prism should be apparent.</p>	<p>(E) Write a computer program that can be used to calculate the volume of right prisms and cylinders.</p> <p>(E) Explore how changing one characteristic of a prism or cylinder changes its volume. For example: A cylinder with a height of 6 cm and a radius of 5 cm is changed to a cylinder with a radius of 10 cm. What happens to the volume?</p> <p>Problem Solving – Apply strategies to solving non-routine problems.</p> <p>e.g., A solid cube is painted red on all sides. The cube is then cut into 27 equal smaller cubes. How many of the smaller cubes have red paint on only 2 sides? (Problem-Solving Challenge for Mathematics, Edmonton: Alberta Education, 1985, p. 47.)</p>		

DATA MANAGEMENT

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>Whenever possible, the objectives of this strand may be integrated throughout the program. The effect is to make the objectives more meaningful than when taught in isolation.</p> <ol style="list-style-type: none"> Applies and practises problem-solving skills in new situations. Maintains previously developed skills (understands purpose, use and misuse of statistics; biases in surveys; represents data in the form of pictographs, bar graphs, line graphs, circle graphs). 	<p>See Grade 8 Data Management, Objectives #2, 4.</p>	<p>(E) Watch a newscast on TV to determine when, where, how and why statistics are used. Discuss their accuracy and timeliness.</p> <p>e.g.,</p> <p>collect, organize and interpret data; make decisions; estimate; predict; draw inferences.</p>	<p>Problem-solving strategies and skills can be a natural outgrowth of a class survey.</p>

DATA MANAGEMENT

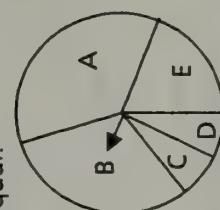
Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3. Analyzes and interprets arguments or conclusions based on statistical information.	<p>Have students research statistical information from magazines and newspapers.</p> <p>Discuss</p>	<ol style="list-style-type: none"> 1. Is the result typical of the population? 2. Is the survey large enough to make a prediction? 3. Are there any noticeable scale variations (e.g., bar graphs without a zero value)? 4. Is there any misrepresented data, incomplete data, or misleading data? (See Holtmath 9 Teacher's Edition, pp. 348-351.) 	<p>(E) Compare the measures of central tendency (mean, mode, median, range). Determine which is most appropriate within its set of data.</p> <p>In using quiz scores (such as 18, 28, 27, 19, 24, 40, 27, 35, 27, 39, 28, 40, 25, 36, 33, 27, 27) determine the measures of central tendency:</p> <ol style="list-style-type: none"> a) mean (average) b) range of values c) median (middle value) d) mode (most frequent value). <p>Discuss when and why each of these measures may be used.</p>
4. In data from meaningful situations (e.g., test marks), understands and uses the terms mean, median, mode and range.			<p>Houghton Mifflin MAC 8, Three M's.</p> <p>Students can design a simple computer program to calculate the mean and possibly the mode, median and range.</p>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>4. In data from meaningful situations (e.g., test marks), understands and uses the terms mean, median, mode and range. (cont'd)</p> <p>5. Distinguishes between a percent and a percentile.</p>	<p>The intent is to convey the meaning of percent and percentile in real-life situations.</p> <p>Define percent as being a fraction out of 100 and percentile as a ranking (from 1 to 99).</p> <p>Use examples to show the difference.</p> <p>e.g., When born, a baby had a mass of 5 kg. Later, the health nurse said that the baby weighed 8 kg (an increase of 60% in mass) and was in the lower 20th percentile of mass for babies of that age.</p>	<p>(E) Find the effect on mean, median, mode and range if:</p> <ol style="list-style-type: none"> a very high score is added the frequency of the lowest value is increased several times more than one value is most frequent. <p>(R) Develop a chart based on test scores. Predict what percentile various scores would have.</p> <p>(E) Conduct a survey on height or weight of people and construct a table. Use the table to determine in what percentile people who were not surveyed would be.</p> <p>Compare percents from test scores to the percentiles from the same tests. Students are not expected to calculate the percentiles but should understand them as rankings.</p>	

DATA MANAGEMENT

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
6. Conducts a survey or poll using correct sampling techniques and reports results using an appropriate table, chart and/or graph.	Decide on what would be a typical sample group. Conduct a survey and choose the best form for displaying the results. Did you use an unlimited response, such as, what is your favourite sweater colour?	<p>(R) Use a restricted choice survey (e.g., What is your favourite dessert – strawberry jello; banana, butterscotch pudding; chocolate ice cream?) and decide on the target group.</p> <p>(E) Conduct a glasses survey: Who wears eyeglasses, contacts, sunglasses, combinations, and who doesn't?</p> <p>How would one answer the question of who wears glasses and misuse the information?</p> <p>Was the survey based on age groups, income groups, etc?</p>	<p>Students design a spinner that has 5 possible outcomes but the sectors are not equal.</p> 
7. Understands and uses the term probability.	Introduce the idea of the impossible and the certain event.	<p>e.g., A person can hold his breath for 1 hour. The sun will rise tomorrow morning.</p> <p>Explore a more systematic approach to probability.</p>	

DATA MANAGEMENT

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>7. Understands and uses the term probability. (cont'd)</p> <p>8. Expresses the probability of the occurrence of an event from a practical situation or a simple experiment or simulation (e.g., pulling a particular coloured marble out of a bag or socks out of a drawer).</p>	<p>Probability can be defined as the chance of an event occurring. This can be calculated by writing a ratio of "the number of outcomes in the event, divided by the total number of possible outcomes".</p> <p>This can be discussed in terms of the probability of a coin toss coming up heads or of a dice roll to produce a certain number.</p>	<p>The student will predict probabilities and then conduct an experiment such as a coin toss to determine an outcome. Have each student toss a coin 10 times. Record and analyze results. Then, compile the class result and analyze them. Compare the two outcomes and discuss the validity and how close the results are to the predicted probability. Other examples may include rolling a dice, drawing a card from a deck of cards, or pulling a coloured marble from a bag of marbles.</p> <p>(E) Students will predict probabilities and then conduct an experiment repeatedly rolling 2 dice. Record the results and determine how close they are to the predicted probabilities.</p> <p>(E) Predict the probability of drawing two aces in a row (or getting consecutive 7's with two dice) and then conduct an experiment to test the predictions.</p>	<p>Students then estimate the probabilities of each outcome. Then they take several spins to see how close they came to the actual probability.</p> <p>From Journeys in Math 7 TRM.</p> <p>Houghton Mifflin MAC 8 Software Probable Urnings. Computer programs that will simulate coin tosses are available. These programs may be used instead of actual experiments.</p> <p>Write a BASIC program to generate random numbers from one to six and then tabulate how many times each number appears.</p> <p>Predict probability with non-standard dice.</p>

DATA MANAGEMENT

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
8. Expresses the probability of the occurrence of an event from a practical situation or a simple experiment or simulation (e.g., pulling a particular coloured marble out of a bag or socks out of a drawer). (cont'd)			<p>e.g.,</p> <pre> 5 REM Random Numbers for the Apple 10 DIM N(6) 20 HOME 30 INPUT "How many numbers do you want?", X 40 FOR Y = 1 TO X 50 A = INT(RND(1)*6 + 1) 60 FOR Z = 1 TO 6 70 IF A = Z THEN N(Z) = N(Z) + 1:GOTO 90 80 NEXT Z 90 NEXT Y 100 PRINT "Final Results" 110 FOR Y = 1 TO 6 120 PRINT Y, N(Y) 130 NEXT 140 END </pre>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>1. Applies and practises problem-solving skills in new situations.</p> <p>2. Maintains previously developed skills (variables; like terms, evaluation of expressions; solving equations; generating and plotting ordered pairs from a given relation).</p> <p>3. Uses formal procedures to solve equations (using all forms of rationals) of the form:</p> $x + a = b, ax = b, \\ ax + b = c, \text{ and } \frac{x}{a} = \frac{b}{c}$ $ax + b = cx, a(x + b) = c \\ \text{and } ax + b = cx + d$	<p>All the ideas and steps involved in general problem solving should be applied to "real life" situations when possible.</p>	<p>Simple graphing programs can be used to generate ordered pairs and plot these pairs on the screen.</p>	

ALGEBRA**Grade 9**

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
4. Verifies solutions to equations.	<p>It is assumed that students can solve equations formally.</p> <p>They should be encouraged to substitute the answer in the original equation and decide if the answer is correct (see Grade 8 Algebra, Objective #5).</p>	(R) Use manipulatives such as a balance of scale to reinforce the concept of equality.	Use of a calculator is encouraged.
5. Manipulates a given formula to change the subject of the formula: e.g., given $x = \frac{f}{w}$ then $w = \frac{f}{x}$	<p>The few formulas presented in mathematics classes are usually taught as a means to an end (e.g., $d = rt$ is used to find distance, rate or time). The result is that little transfer occurs to knowing how to use other formulas in mathematics and other disciplines.</p> <p>A much more holistic view of formulas must be taken. In Grade 8, the outcome of objective #6 (Algebra) is that students will understand relationships among the variables, recognize the similarities (or differences) among those relationships (regardless of what kind of variables are used) and will know how to substitute and solve for missing elements.</p>	<p>Computers and calculators may be used to check some of the values on the chart.</p>	The intent of this objective is to provide students with an alternative strategy for finding the missing value in a formula, especially in situations for which the required missing element is always the same.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>5. Manipulates a given formula to change the subject of the formula: (cont'd)</p> <p>e.g., given $x = \frac{f}{w}$ then $w = \frac{f}{x}$</p> <p>For example, if students were required to find the principals in a number of loans, manipulating the formula first (using $P = i/t$) is much more convenient than using the form $i = prt$ (and then having to substitute and isolate the p). Review the relationships among the variables in any formula (real or contrived) and demonstrate that the techniques for isolating a particular variable are the same whether the variable has been replaced with a number or not.</p> <p>(E) Discuss the effect on a selected element of a formula if another element is doubled, tripled, etc. e.g., What happens to A, in $A = \pi r^2$, if r is doubled? tripled?</p>			

Objective	Clarification or Example	Elective Suggestions Integration of Problem Solving and Technology																					
5. Manipulates a given formula to change the subject of the formula: (cont'd)	<p>A formula which can be practically verified is $D = \frac{M}{V}$</p> <p>where D means Density M means Mass V means Volume</p> <p>Have the students weigh and measure the volume of wooden blocks of different sizes (but same type of wood).</p> 	<table border="1" data-bbox="825 1183 1274 1581"> <thead> <tr> <th data-bbox="825 1183 888 1581">M</th> <th data-bbox="888 1183 951 1581">V</th> <th data-bbox="951 1183 1014 1581">$D = \frac{M}{V}$</th> </tr> </thead> <tbody> <tr><td data-bbox="825 1581 888 1623">1</td><td data-bbox="888 1581 951 1623"></td><td data-bbox="951 1581 1014 1623"></td></tr> <tr><td data-bbox="825 1623 888 1665">2</td><td data-bbox="888 1623 951 1665"></td><td data-bbox="951 1623 1014 1665"></td></tr> <tr><td data-bbox="825 1665 888 1707">3</td><td data-bbox="888 1665 951 1707"></td><td data-bbox="951 1665 1014 1707"></td></tr> <tr><td data-bbox="825 1707 888 1749">4</td><td data-bbox="888 1707 951 1749"></td><td data-bbox="951 1707 1014 1749"></td></tr> <tr><td data-bbox="825 1749 888 1791">5</td><td data-bbox="888 1749 951 1791"></td><td data-bbox="951 1749 1014 1791"></td></tr> <tr><td data-bbox="825 1791 888 1833">6</td><td data-bbox="888 1791 951 1833"></td><td data-bbox="951 1791 1014 1833"></td></tr> </tbody> </table> <p>When the chart is filled, verify the values using $M = DV$ or $D = \frac{M}{V}$</p>	M	V	$D = \frac{M}{V}$	1			2			3			4			5			6		
M	V	$D = \frac{M}{V}$																					
1																							
2																							
3																							
4																							
5																							
6																							

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology									
<p>6. Finds a missing element of a formula through manipulation.</p> <p>7. Solves inequalities of the form $x + a \geq b$ and $cx \leq d$ (c is positive; direction of inequalities may vary).</p> <p>8. Verifies solutions to inequalities.</p> <p>This objective is an extension of the previous objective (#5). The development of objective #5 should be extended to include actual computations of values.</p> <p>(See Holtmath 9 Teacher's Edition, p. 143.)</p> <p>Problems in which a maximum or minimum value may be explored.</p> <p>To verify inequalities, it is necessary to determine the limits on boundaries. Substitute the maximum or minimum value.</p> <p>e.g.,</p> <table style="margin-left: 200px;"><tr><td>$3x > 9$</td><td>$x > 3$</td></tr><tr><td>Use</td><td>$x = 3$</td></tr><tr><td>L.S.</td><td>$= 3(3)$</td></tr><tr><td></td><td>$= 9$</td></tr><tr><td>R.S.</td><td>$= 9$</td></tr><tr><td>L.S.</td><td>$= R.S.$</td></tr></table> <p>The limits have been found to be correct.</p>	$3x > 9$	$x > 3$	Use	$x = 3$	L.S.	$= 3(3)$		$= 9$	R.S.	$= 9$	L.S.	$= R.S.$
$3x > 9$	$x > 3$											
Use	$x = 3$											
L.S.	$= 3(3)$											
	$= 9$											
R.S.	$= 9$											
L.S.	$= R.S.$											

ALGEBRA

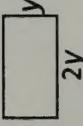
Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>9. Graphs solutions to inequalities on a number line.</p>	<p>Reviews the meaning of the inequality symbols $<$ and $>$ and the combined symbols of \leq and \geq. A method for plotting or a number line should be decided upon; i.e., "is more than" is the same as $>$ and can use a circle () whereas "is at least" is the same as \geq and can use a dot. ()</p> <p>(See Holtmath 9 Teacher's Edition, pp. 143-145.)</p>	<p>Draw graphs in which the values of x and y are between certain limits (domain and range respectively). Have the students graph the domain and range on a number line.</p>	<p>Houghton Mifflin, Mathematics Activities Software, Grade 8, pp. 66-68.</p> <p>Explore problems in which patterns develop, using logic or reasoning, generalities solutions.</p> <p>(E) Solve simultaneous linear relations by graphing and determining the point of intersection.</p>
	<p>10. Given a set of ordered pairs or a table of values, writes the function that determines the relation (limit: linear relations).</p>	<p>From a table of values students try to determine the relation by looking at the pattern or using guess and check. Relations should be limited to the form $y = ax + b$. (Plotting ordered pairs is a maintenance skill but may be reviewed here).</p>	<p>Have students make up their own tables from a relation. Exchange tables for others to determine the relation form.</p> <p>(E) Explore how changing a or b in a linear relation of the form $y = ax + b$, changes the graph. This can be extended to students making predictions concerning the direction of the line and the steepness of the line as a or b changes.</p>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>1. Knows the terms associated with polynomials: monomial, binomial, trinomial, degree, numerical and literal coefficient.</p> <p>2. Find the sum and difference of monomials.</p>	<p>(Holtmath 9 Teacher's Edition, pp. 256-257.)</p> <p>Use manipulatives in the form of squares and rectangles to develop the concept of "like" terms.</p> <p>Use <input type="checkbox"/> and <input type="checkbox"/> to represent the squares and rectangles. <input type="checkbox"/> and <input type="checkbox"/> are like terms. Therefore, <input type="checkbox"/> + <input type="checkbox"/> = <input type="checkbox"/> = 4 <input type="checkbox"/></p> <p>Finally use $3x$ and $4x$ as examples of like terms and $3x$ and $5y$ as unlike terms. $3x + 4x = 7x$. Conclude that to add or subtract like terms one must add or subtract the numerical coefficients. Remember to subtract, add the opposite.</p> $ \begin{aligned} 4x - (-2x) \\ = 4x + 2x \\ = 6x \end{aligned} $		

ALGEBRA-EXTENDED CONTENT

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>3. Finds the product and quotient of monomials.</p>	<p>(MULTIPLYING MONOMIALS)</p> <p>Give some numerical examples of products of the same factors, e.g.,</p> $4 \times 4 = 4^2$ $4 \times 4 \times 4 = 4^3$ $4 \times 4 \times 4 \times 4 = 4^4$ <p>Use the same type of examples with letters instead of numbers</p> $y \times y = y^2$ $y \times y \times y = y^3$ $y \times y \times y \times y = y^4$	<p>Use the Diskette: Sunburst Communications – Polynomial Practice Using Tiles.</p> <p>e.g., $(2) \times (3) = 6$ square units</p>  <p>y y $(y)(y) = y^2$</p>  <p>$(2y)(y) = 2y^2$</p>	

ALGEBRA-EXTENDED CONTENT

Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
<p>3. Finds the product and quotient of monomials. (cont'd)</p> <p>Manipulatives to be used for the area concept may be cardboard cut out in rectangular shapes. After this introduce abstract examples.</p> <p>e.g., a. 5×4 b. $2x \times 3$ c. $(4x) \times (3x)$</p> <p>Using a rectangle, note the calculation of area using sides of $4a$ and $3b$. To multiply two or more monomials, multiply the numerical coefficients and multiply the literal coefficients.</p> <p>e.g., $(4a)(3b) = 12ab$</p>	<p>$4a$ </p> <p>Using examples with numbers (i.e., $2^5 \div 2^2$) give the rule for division of powers with the same base. To divide monomials you divide the numerical coefficients and subtract the exponents on the same bases. i.e., $2^5 \div 2^2 = 2^{5-2} = 2^3$ $x^5 \div x^2 = x^{5-2} = x^3$</p>		

QA 14 C22 A3 A29 1987
JUNIOR HIGH MATHEMATICS
CURRICULUM GUIDE 1987 DRAFT/

39908342 CURR HIST



000022796635

QA 14 C22 A3 A29 1987
Junior high mathematics
curriculum guide 1987 :

39908342 CURR HIST

CURRICULUM GUIDE

For Reference

NOT TO BE TAKEN FROM THIS ROOM

EXCEPT WITH LIBRARIAN'S PERMISSION

